

# CONTENTS

	Page	
<u>SUMMARY</u> . . . . .	1	1/A8
<u>INTRODUCTION</u> . . . . .	1	1/A8
<u>ANALYSIS</u> . . . . .	2	1/A9
SOLUTION PROCEDURES FOR PROBLEM I . . . . .	4	1/A11
Procedure I-C1 . . . . .	5	1/A12
Procedure I-C2 . . . . .	5	1/A12
Procedure I-C3 . . . . .	7	1/A14
Procedure I-C4 . . . . .	8	1/B1
Procedure I-A1 . . . . .	9	1/B2
Procedure I-A2 . . . . .	10	1/B3
Procedure I-A3 . . . . .	12	1/B5
Procedure I-A4 . . . . .	13	1/B6
SOLUTION PROCEDURES FOR PROBLEM II . . . . .	14	1/B7
Procedure II-C1 . . . . .	15	1/B8
Procedure II-C2 . . . . .	15	1/B8
Procedure II-C3 . . . . .	16	1/B9
Procedure II-C4 . . . . .	17	1/B10
Procedure II-A1 . . . . .	19	1/B12
Procedure II-A2 . . . . .	20	1/B13
Procedure II-A3 . . . . .	21	1/B14
Procedure II-A4 . . . . .	22	1/C1
SOLUTION PROCEDURES FOR PROBLEM III . . . . .	24	1/C3
Procedure III-C1 . . . . .	24	1/C3
Procedure III-A1 . . . . .	25	1/C4
<u>DISCUSSION</u> . . . . .	25	1/C4
CRITICAL ASSUMPTIONS AND COMPARISON OF EXAMPLE-		
CASE SOLUTIONS . . . . .	25	1/C4
Cruise Mean Level . . . . .	26	1/C5
Log-Normal Distribution . . . . .	26	1/C5
Retention Ratio . . . . .	27	1/C6
Maximum Values . . . . .	28	1/C7
CALCULATIONS USING TABULATED AMBIENT OZONE DATA . . . . .	29	1/C8
EFFECT ON PROBABILITY ESTIMATES OF VARYING LATITUDE,		
ALTITUDE, SEASON, RETENTION RATIO, FLIGHT DURATION, AND		
CABIN OZONE LIMITS . . . . .	31	1/C10

SUMMARY OF RESULTS . . . . . 32 1/C11

APPENDIXES

    A - PROPOSED RULES . . . . . 34 1/C13

    B - EXAMPLE-CASE DATA SET SUMMARY . . . . . 35 1/C14

REFERENCES . . . . . 36 1/D1

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**Procedures for Estimating  
the Frequency of Commercial  
Airline Flights Encountering  
High Cabin Ozone Levels**

**James D. Holdeman**

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**NASA Technical Paper 1560**

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**James D. Holdeman**  
*Lewis Research Center*  
*Cleveland, Ohio*



National Aeronautics  
and Space Administration

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# CONTENTS

	Page
<u>SUMMARY</u> . . . . .	1
<u>INTRODUCTION</u> . . . . .	1
<u>ANALYSIS</u> . . . . .	2
SOLUTION PROCEDURES FOR PROBLEM I . . . . .	4
Procedure I-C1 . . . . .	5
Procedure I-C2 . . . . .	5
Procedure I-C3 . . . . .	7
Procedure I-C4 . . . . .	8
Procedure I-A1 . . . . .	9
Procedure I-A2 . . . . .	10
Procedure I-A3 . . . . .	12
Procedure I-A4 . . . . .	13
SOLUTION PROCEDURES FOR PROBLEM II . . . . .	14
Procedure II-C1 . . . . .	15
Procedure II-C2 . . . . .	15
Procedure II-C3 . . . . .	16
Procedure II-C4 . . . . .	17
Procedure II-A1 . . . . .	19
Procedure II-A2 . . . . .	20
Procedure II-A3 . . . . .	21
Procedure II-A4 . . . . .	22
SOLUTION PROCEDURES FOR PROBLEM III . . . . .	24
Procedure III-C1 . . . . .	24
Procedure III-A1 . . . . .	25
<u>DISCUSSION</u> . . . . .	25
CRITICAL ASSUMPTIONS AND COMPARISON OF EXAMPLE-	
CASE SOLUTIONS . . . . .	25
Cruise Mean Level . . . . .	26
Log-Normal Distribution . . . . .	26
Retention Ratio . . . . .	27
Maximum Values . . . . .	28
CALCULATIONS USING TABULATED AMBIENT OZONE DATA . . . . .	29
EFFECT ON PROBABILITY ESTIMATES OF VARYING LATITUDE,	
ALTITUDE, SEASON, RETENTION RATIO, FLIGHT DURATION, AND	
CABIN OZONE LIMITS . . . . .	31

<u>SUMMARY OF RESULTS</u> . . . . .	32
<u>APPENDIXES</u>	
A - PROPOSED RULES . . . . .	34
B - EXAMPLE-CASE DATA SET SUMMARY . . . . .	35
<u>REFERENCES</u> . . . . .	36

## SUMMARY

This report presents and discusses several procedures for estimating the frequency at which commercial airline flights will encounter high cabin ozone levels. This work was motivated by rules proposed by the Federal Aviation Administration that, if adopted, would establish limits on ozone levels in commercial airliner cabins and would require air carriers to demonstrate, by analysis or tests, that their aircraft would be in compliance with the rules.

Based on the proposed rules, three analytical problems are formulated, with solution procedures addressed to different levels of input information - from complete cabin ozone measurements, which provide the required estimate directly, to limited ozone information, such as ambient ozone means and standard deviations, with which several assumptions are necessary to obtain the required estimate. Solutions for each problem are compared by using a set of simultaneous cabin and ambient ozone data, obtained by the NASA Global Atmospheric Sampling Program (GASP), for a Boeing 747-100 airliner in domestic U.S. service from March 30, 1977, to June 13, 1977. Example calculations are also performed by using tabulated ambient ozone data available in the literature to illustrate how variations in latitude, altitude, season, retention ratio, flight duration, and cabin ozone limits affect the estimated probabilities.

## INTRODUCTION

Since March 1975, the NASA Global Atmospheric Sampling Program (GASP) has been obtaining, archiving, and analyzing atmospheric trace-constituent data in the upper troposphere and lower stratosphere (ref. 1 and references therein). These data are acquired by fully automated sampling systems on several Boeing 747 aircraft in routine commercial service. The objectives of the program are (1) to provide a better understanding of the composition and dynamics of the atmosphere at the altitudes where commercial aircraft fly and (2) to provide initial-value boundary conditions for atmospheric models being used to assess potentially adverse effects from aircraft exhaust emissions on the natural atmosphere.

In addition to the ambient atmospheric constituent measurements, GASP began, in March 1977, to measure ozone levels in the cabins of a 747-100 and a 747SP aircraft. These measurements are providing simultaneous cabin and ambient ozone data

on flights of varying duration and at different flight levels, geographical locations, and seasons. These measurements came about when, in the winter of 1976-77, the airlines received complaints about passenger and crewmember discomfort during flight. Since it is well known that subsonic commercial aircraft sometimes cruise in the stratosphere, where ambient ozone levels increase rapidly with altitude, and that the frequency of flights in this region is greatest at mid-to-high (northerly) latitudes in the winter and spring, when the tropopause is lowest, ozone was suspected as the cause of the reported discomfort.

Based on Occupational Safety and Health Administration (OSHA) ozone standards and analyses of the available data (including GASP ambient ozone measurements (refs. 2 to 4) and simultaneous cabin and ambient ozone measurements from selected GASP flights (refs. 4 and 5), the Federal Aviation Administration (FAA) has issued a Notice of Proposed Rulemaking regarding acceptable levels of ozone in aircraft cabins (ref. 6). A summary of the proposed rules is given in appendix A. In addition to establishing limits, these rules would require that compliance "must be shown by analysis or tests based on airplane operational procedures and performance limitations." Thus, the proposed rules clearly establish the need for both government and industry to reliably estimate whether (or when) any given aircraft would require hardware or flight route modifications to meet the regulation, if adopted.

This report outlines several procedures for estimating the frequency of flights encountering high cabin ozone levels. Three analytical problems are considered: namely, estimating flight-segment-mean levels, estimating maximum-per-flight levels, and estimating mean levels over any 2-hour interval. Several solution procedures, appropriate to different levels of input information, are given for each problem. Each procedure is illustrated by an example-case calculation. Critical assumptions are discussed and evaluated, and the several solutions for each problem are compared.

## ANALYSES

In this section, procedures for solving the analytical problems posed by the proposed rules are outlined. Three problems are considered:

Problem I - For a given aircraft and routes, estimate the fraction of flights on which a specified time-weighted, flight-segment-average cabin ozone level would be exceeded (appendix A, article 121.578 (b)).

Problem II - For a given aircraft and routes, estimate the fraction of flights on which a specified maximum cabin ozone level would be exceeded (appendix A, articles 25.832 (a) and 121.578 (a)).

Problem III - For a given aircraft and routes, estimate the fraction of flights on which a specified maximum 2-hour-average cabin ozone level would be exceeded (appendix A, article 25.832 (b)).

The procedures for each problem vary from the direct solution, when appropriate cabin ozone data are available, to very indirect solutions based on more limited information, as would be the case for noninstrumented aircraft. Limited data, of course, require that some assumptions be made, and the accuracy of the estimate will depend on the validity of these assumptions.

Although it is not within the scope of the present report to determine whether (or when) particular aircraft types or routes would be subject to high cabin ozone encounters, an example case is given with the analysis to illustrate the procedures. The data used for this example are from a Boeing 747-100 airliner in domestic U.S. service from March 30, 1977, to June 13, 1977 (GASP tape VL0012, file 2) and are restricted to observations in which simultaneous cabin and ambient ozone measurements were made. For this case, since complete cabin ozone data are available, the problems can be solved directly. The alternative procedures are given to show estimates that might be made from more limited information. Although the example-case calculations use the specific limits proposed in reference 6, the procedures are appropriate over the range of possible limits.

The solution procedures for each problem either use, or result in, curves called cumulative frequency distributions (cfd's). These curves show the fraction of flights (or observations) for which the ozone level would exceed or equal any given ozone level. Ambient ozone cfd's vary with altitude, season, and routing (ref. 4) - just as the mean ambient ozone level varies with altitude, season, and geographical location. In addition, cabin ozone cfd's can vary between aircraft types, even for the same routes and dates, if the fraction of the ambient ozone that enters the cabin is different. Figures 1 to 5 are cfd's for the example-case solutions in the following sections.

Since several procedures are given for each problem, they are identified so that the problem to which they apply and the known input information are readily apparent. The Roman numeral assigned to each procedure identifies the problem being solved; the letter identifies whether the procedure uses cabin (C) or ambient (A) ozone data; and the following Arabic numeral identifies the completeness of the ozone data used; that is,

- 1 mean-per-flight, maximum-per-flight, or maximum 2-hour ozone data for a series of flights
- 2 the mean and the standard deviation of mean-per-flight or maximum-per-flight ozone data



3 ozone data from a series of observations

4 mean and standard-deviation ozone values from a series of observations

Data of the first two types would have to come from aircraft measurements. Data of the second two types could be aircraft obtained or, for procedures using ambient ozone, could be derived from ozonesonde data or other climatological (historical) compilations (see the section DISCUSSION).

The required input information for each procedure and the estimated probabilities for the example-case solutions are summarized for problems I to III in tables I to III, respectively. A summary tabulation of the example-case data set is given in table IV; the contents and format of this table are discussed in appendix B. All the cfd's for the example-case solutions are derivable from the data given in table IV, except for those cfd's based on all the observations. For completeness, the data for these curves are given in table V.

#### SOLUTION PROCEDURES FOR PROBLEM I

For a given aircraft type and given routes (latitude, altitude, and date ranges), estimate the fraction of flights on which a specified time-weighted, flight-segment-average ozone level would be exceeded (appendix A, article 121.578 (b)).

The time-weighted, flight-segment-average limit applies from takeoff to touchdown. Since the available data (whether from GASP or ozonesondes) are at cruise altitudes, the specified limit must be adjusted to cruise conditions. This is done by assuming a time increment for ascent and descent for each flight and by assuming a typical tropospheric ozone level for this period. Let

FLTLMT specified flight-segment-average cabin ozone limit

UPDOWN ascent-plus-descent time increment

TROP assumed cabin ozone level during UPDOWN

TDATA time at cruise

LIMIT mean cabin ozone limit during TDATA

Then

$$\text{LIMIT} = (\text{FLTLMT}) \left( \frac{(\text{TDATA} + \text{UPDOWN})}{\text{TDATA}} - \frac{(\text{TROP}) (\text{UPDOWN})}{\text{TDATA}} \right) \quad (1)$$

For the example-case data (table IV), the average data duration per flight TDATA is 3.35 hours. From a comparison of the TDATA times in table IV with the scheduled flight times, we assume that UPDOWN equals 1 hour. From references 2 and 3, we choose an ozone level of 0.05 ppmv to be representative of tropospheric conditions; thus TROP is 0.05 ppmv.

The required input information and the estimated probabilities for the example-case solutions to problem I are summarized for each procedure in table I.

#### Procedure I-C1

Given: Cruise-mean cabin ozone levels for a series of flights (mean cabin ozone per-flight cfd)

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Obtain directly the fraction of flights on which the time-weighted-average cabin ozone level at cruise exceeds LIMIT.

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) From table IV, for a LIMIT of 0.115 ppmv,

$$P = \frac{17}{70} = 0.243$$

(fig. 1, curve A, for a cabin ozone level of 0.115 ppmv).

#### Procedure I-C2

Given: Cabin ozone cruise-mean mean and standard deviation for a series of flights

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Since only the cabin ozone cruise-mean mean and standard deviation are known, the form of the cabin ozone frequency distribution must be assumed. In this procedure we have assumed a log-normal distribution; the appropriateness of this assumption is examined in the section DISCUSSION.

Assume that the frequency distribution of the cruise-mean data is log-normal, and calculate the parameters  $\mu$  and  $\sigma$  from the given mean  $M$  and the given standard deviation  $SD$  (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$

- (3) Determine the probability that the cruise-mean cabin ozone level exceeds LIMIT on any flight, from the standardized normal distribution, as follows:
  - (a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{LIMIT}) - \mu}{\sigma}$$

- (b) Obtain the probability that the cruise-mean cabin ozone level exceeds LIMIT on any flight, from the cumulative density function  $\theta$  (e.g., ref. 8).

$$P = 1 - \theta(z)$$

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For an  $M$  of 0.095 ppmv and an  $SD$  of 0.058 ppmv (calculated from cruise-mean cabin ozone values in table IV),



$$\mu = \ln(0.095) - 0.5 \ln \left[ \left( \frac{0.058}{0.095} \right)^2 + 1 \right] = -2.515$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.058}{0.095} \right)^2 + 1 \right]} = 0.586$$

(3) For a LIMIT of 0.115 ppmv,

$$(a) z = \frac{\ln(0.115) - (-2.515)}{0.586} = 0.622$$

$$(b) P = 1 - \theta(0.622) = 0.267$$

(fig. 1, curve B, for a cabin ozone level of 0.115 ppmv).

### Procedure I-C3

Given: Cabin ozone levels for a series of observations (cabin ozone cfd)

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Obtain the fraction of observations that exceed LIMIT from the cfd.
- (3) To proceed, we must transform data from a series of observations to average data per flight. In this regard, we assume that the probability that the cruise-mean cabin ozone level exceeds LIMIT is the same as the probability that the cabin ozone level exceeds LIMIT on any observation. The appropriateness of this assumption is examined in the section DISCUSSION.

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) From table V, for a LIMIT of 0.115 ppmv,

$$p = 0.230$$

(fig. 1, curve C, for a cabin ozone level of 0.115 ppmv).

- (3)  $P = 0.230$

#### Procedure I-C4

Given: Cabin ozone mean and standard deviation for a series of observations

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Since only the cabin ozone mean and standard deviation are known, the form of the frequency distribution of cabin ozone must be assumed. In this procedure we have assumed a log-normal distribution: the appropriateness of this assumption is examined in the section DISCUSSION.

Assume that the frequency distribution of the cabin ozone measurements is log-normal, and calculate the parameters  $\mu$  and  $\sigma$  from the given mean  $M$  and the given standard deviation  $SD$  (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$

- (3) Determine the probability that the cabin ozone level exceeds LIMIT on any observation, from the standardized normal distribution, as follows:
  - (a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{LIMIT}) - \mu}{\sigma}$$

- (b) Obtain the probability that LIMIT is exceeded on any observation from the cumulative density function  $\theta$  (e.g., ref. 8).

$$p = 1 - \theta(z)$$

- (4) Assume that the probability that the cruise-mean cabin ozone level exceeds LIMIT is the same as the probability that the cabin ozone level exceeds LIMIT on any observation (see the section DISCUSSION).

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For an M of 0.090 ppmv and an SD of 0.075 ppmv (all observations - from table IV),

$$\mu = \ln(0.090) - 0.5 \ln \left[ \left( \frac{0.075}{0.090} \right)^2 + 1 \right] = -2.672$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.076}{0.090} \right)^2 + 1 \right]} = 0.726$$

- (3) For a LIMIT of 0.115 ppmv,

$$(a) z = \frac{\ln(0.115) - (-2.672)}{(0.726)} = 0.701$$

$$(b) p = 1 - \theta(0.701) = 0.242$$

(fig. 1, curve D, for a cabin ozone level of 0.115 ppmv).

- (4) P = 0.242

In procedures I-C1, I-C2, I-C3, and I-C4, cabin ozone data were used. Procedures I-A1, I-A2, I-A3, and I-A4, which follow, are parallel, respectively, to procedures I-C1, I-C2, I-C3, and I-C4, except that ambient ozone data are used. In these cases, the retention ratio (ratio of cabin ozone to ambient ozone) for the aircraft must be known or assumed.

This requirement is a critical step in the procedures, as the retention ratio varies appreciably between aircraft types (e.g., refs. 1, 4, and 5). Also, for simplicity in the calculations, it has been assumed that the retention ratio is constant for a given aircraft type, although there is evidence that this ratio varies with flight duration and ambient ozone level (see the section DISCUSSION).

#### Procedure I-A1

Given: Cruise-mean ambient ozone levels for a series of flights (mean ambient ozone per-flight cfd) and the retention ratio

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).

- (2) Determine the maximum allowable cruise-mean ambient ozone level from the cabin ozone cruise-mean limit (LIMIT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{LIMIT}}{r}$$

- (3) Then obtain the fraction of flights on which the cruise-mean ambient ozone level exceeds AMBLMT from the cfd.

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hour, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For a LIMIT of 0.115 ppmv and  $r$  of 0.465 (from table IV),

$$\text{AMBLMT} = \frac{0.115}{0.465} = 0.247 \text{ ppmv}$$

- (3) From table IV, for an AMBLMT of 0.247 ppmv,

$$P = \frac{17}{70} = 0.243$$

(fig. 2, curve E, for an ambient ozone level of 0.247 ppmv).

Procedure I-A2

Given: Ambient ozone cruise-mean mean and standard deviation for a series of flights and the retention ratio

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to a specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Determine the maximum allowable cruise-mean ambient ozone level from the cabin ozone cruise-mean limit (LIMIT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{LIMIT}}{r}$$

- (3) Assume that the frequency distribution of the ambient zone cruise-mean data is log-normal (see the section DISCUSSION), and calculate the parameters  $\mu$  and  $\sigma$  from the given mean  $M$  and the given standard deviation  $SD$  (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$

- (4) Determine the probability that the cruise-mean ambient ozone level exceeds AMBLMT on any flight, from the standardized normal distribution, as follows:
- (a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{AMBLMT}) - \mu}{\sigma}$$

- (b) Obtain the probability that AMBLMT is exceeded on any flight from the cumulative density function  $\theta$  (e.g., ref. 8).

$$P = 1 - \theta(z)$$

#### Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For a LIMIT of 0.115 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.115}{0.465} = 0.247 \text{ ppmv}$$

- (3) For an  $M$  of 0.203 ppmv and an  $SD$  of 0.145 ppmv (calculated from cruise-mean ambient-ozone values in table IV),

$$\mu = \ln(0.203) - 0.5 \ln \left[ \left( \frac{0.145}{0.203} \right)^2 + 1 \right] = -1.798$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.145}{0.203} \right)^2 + 1 \right]} = 0.640$$

(4) For an AMBLMT of 0.247 ppmv,

$$(a) z = \ln \frac{(0.247 - (-1.798))}{(0.640)} = 0.625$$

$$(b) P = 1 - \theta(0.625) = 0.266$$

(fig. 2, curve F, for an ambient ozone level of 0.247 ppmv).

#### Procedure I-A3

Given: Ambient ozone levels for a series of observations (ambient ozone cfd) and the retention ratio

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to the specified flight-segment-average cabin ozone limit (FLTLMT) from equation (1).
- (2) Determine the maximum allowable cruise-mean ambient ozone level from the cabin ozone cruise-mean limit (LIMIT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{LIMIT}}{r}$$

- (3) Obtain the fraction of observations for which the ambient ozone level exceeds AMBLMT from the cfd.
- (4) Assume that the probability that the cruise-mean ambient ozone level exceeds AMBLMT is the same as the probability that the ambient ozone level exceeds AMBLMT on any observation (see the section DISCUSSION).

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For a LIMIT of 0.115 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.115}{0.465} = 0.247 \text{ ppmv}$$

(3) From table V, for an AMBLMT of 0.247 ppmv,

$$p = 0.220$$

(fig. 2, curve G, for an ambient ozone level of 0.247 ppmv).

(4)  $P = 0.220$

#### Procedure I-A4

Given: Ambient ozone mean and standard deviation for a series of observations and the retention ratio

Solution:

- (1) Determine the cabin ozone cruise-mean limit (LIMIT) corresponding to the specified flight-segment-average cabin ozone limit (FSLTMT) from equation (1).
- (2) Determine the maximum allowable cruise-mean ambient ozone level from the cabin ozone cruise-mean limit (LIMIT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{LIMIT}}{r}$$

- (3) Assume that the frequency distribution of the ambient ozone measurements is log-normal (see the section DISCUSSION), and calculate the parameters  $\mu$  and  $\sigma$  from the given mean  $M$  and the given standard deviation  $SD$  (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$

- (4) Determine the probability that the ambient ozone level exceeds AMBLMT on any observation, from the standardized normal distribution, as follows:
  - (a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{AMBLMT}) - \mu}{\sigma}$$

- (b) Obtain the probability that AMBLMT is exceeded on any observation from the cumulative density function  $\theta$  (e.g., ref. 8).

$$p = 1 - \theta(z)$$

- (5) Assume that the probability that the cruise-mean ambient ozone level exceeds AMBLMT is the same as the probability that the ambient ozone level exceeds AMBLMT on any observation (see the section DISCUSSION).

Example:

- (1) For an FLTLMT of 0.1 ppmv, a TDATA of 3.35 hours, an UPDOWN of 1 hour, and a TROP of 0.05 ppmv,

$$\text{LIMIT} = (0.1) \frac{(3.35 + 1)}{3.35} - \frac{(0.05)(1)}{3.35} = 0.115 \text{ ppmv}$$

- (2) For a LIMIT of 0.115 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.115}{0.465} = 0.247 \text{ ppmv}$$

- (3) For an  $M$  of 0.191 ppmv and an  $SD$  of 0.185 ppmv (all observations - from table IV),

$$\mu = \ln(0.191) - 0.5 \ln \left[ \left( \frac{0.185}{0.191} \right)^2 + 1 \right] = -1.986$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.185}{0.191} \right)^2 + 1 \right]} = 0.813$$

- (4) For an AMBLMT of 0.247 ppmv,

$$(a) z = \frac{\ln(0.247) - (-1.986)}{(0.813)} = 0.723$$

$$(b) p = 1 - \theta(0.723) = 0.236$$

(fig. 2, curve H, for an ambient ozone level of 0.247 ppmv).

$$(5) P = 0.236$$

## SOLUTION PROCEDURES FOR PROBLEM II

For a given aircraft type and given routes (latitude, altitude, and date ranges), estimate the fraction of flights on which a specified maximum cabin ozone level would



be exceeded (appendix A, articles 25.832 (a) and 121.578 (a)).

The required input information and the estimated probabilities for the example-case solutions to problem II are summarized in table II.

#### Procedure II-C1

Given: Maximum cabin ozone levels for a series of flights (maximum cabin ozone per-flight cfd)

Solution: For any specified cabin ozone limit, say MAXLMT, the fraction of flights on which this level would be exceeded is obtained directly.

Example:

Let MAXLMT equal 0.3 ppmv; then from table IV we obtain

$$P = \frac{14}{70} = 0.20$$

(fig. 3, curve I, for a cabin ozone level of 0.3 ppmv).

#### Procedure II-C2

Given: Maximum cabin ozone mean and standard deviation for a series of flights

- (1) Assume that the cabin ozone maximum-per-flight data can be approximated by an extreme-value distribution (ref. 7), and calculate the parameters  $\mu$  and  $\sigma$  for this distribution from the given mean  $M$  and the given standard deviation  $SD$  as

$$\mu = M - \frac{0.5776 SD}{\sqrt{1.645}}$$

$$\sigma = \frac{SD}{\sqrt{1.645}}$$

- (2) The required probability that the maximum cabin ozone level on any flight will exceed the specified limit (MAXLMT) is given by

$$P = 1 - \exp \left[ -\exp \left( -\frac{\text{MAXLMT} - \mu}{\sigma} \right) \right]$$

Example:

- (1) For an  $M$  of 0.201 ppmv and an  $SD$  of 0.103 ppmv (calculated from the cabin ozone maximum-per-flight data in table IV),

$$\mu = 0.201 - \frac{(0.5776)(0.103)}{\sqrt{1.645}} = 0.155$$

$$\sigma = \frac{0.103}{\sqrt{1.645}} = 0.0803$$

- (2) Then, for an MAXLMT of 0.3 ppmv,

$$P = 1 - \exp \left[ -\exp \left( -\frac{0.3 - 0.155}{0.0803} \right) \right] = 0.152$$

(fig. 3, curve J, for a cabin ozone level of 0.3 ppmv).

Procedure II-C3

Given: Cabin ozone levels for a series of observations (cabin ozone cfd - all observations)

Solution:

- (1) Determine the probability that the cabin ozone level exceeds the specified limit (MAXLMT) on any observation from the cfd. Since the known probability is that of exceeding MAXLMT on any observation, we proceed by determining the number of independent observations  $n$  on an average-duration flight (based on the characteristic scale of ambient ozone variability) and use the binomial formula (e.g., ref. 8) to obtain the probability that MAXLMT will be exceeded on at least one of these  $n$  independent observations (see the section DISCUSSION).
- (2) Determine the number of independent observations  $n$  per flight.
  - (a) Determine the average time at cruise (TDATA).
  - (b) Determine the time between independent observations (DELT).
  - (c) Then  $n = \frac{\text{TDATA}}{\text{DELT}}$
- (3) The probability of encountering exactly  $k$  observations per flight with a cabin ozone level exceeding MAXLMT is

$$P(k) = \frac{n!}{k!(n-k)!} (p)^{(k)}(1-p)^{(n-k)}$$

and the probability of encountering no observations with a cabin ozone level exceeding MAXLMT is

$$P(k \geq 1) = 1 - P(k = 0)$$

(e.g., ref. 8).

#### Example:

- (1) From table V, for an MAXLMT of 0.3 ppmv,

$$p = 0.03$$

(fig. 1, curve C, for a cabin ozone level of 0.3 ppmv).

- (2) TDATA = (234:16)/70 flights = 3.35 hours (table IV). Reference 9 determined the distance between independent ambient zone observations to be 2d = 450 kilometers. Assuming that this is appropriate for cabin ozone also and for an average airspeed of 890 kilometers per hour (480 knots) yields

$$\text{DELT} = \frac{450}{890} = 0.5 \text{ hr}$$

For a TDATA of 3.35 hours and a DELT of 0.5 hour,

$$n = \frac{3.35}{0.5} = 6.7$$

- (3) For an n of 6.7 and a p of 0.03,

$$P(k \geq 1) = 1 - \frac{6.7!}{0!(6.7-0)!} (0.03)^0 (1-0.03)^{6.7} = 0.185$$

(fig. 3, curve K, for a cabin ozone level of 0.3 ppmv).

#### Procedure II-C4

Given: Cabin ozone mean and standard deviation from a series of observations

Solution:

- (1) Assume that the frequency distribution of the cabin ozone measurements is log-normal (see the section DISCUSSION), with a mean M and a standard deviation SD, and calculate the parameters  $\mu$  and  $\sigma$  for the assumed distribution as follows (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$

- (2) Determine the probability that the cabin ozone level exceeds the specified limit (MAXLMT) on any observation, from the standardized normal distribution, as follows:

- (a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{MAXLMT}) - \mu}{\sigma}$$

- (b) Obtain the probability that the cabin ozone level exceeds MAXLMT on any observation from the cumulative density function  $\theta$  (e.g., ref. 8).

$$p = 1 - \theta(z)$$

- (3) Determine the time at cruise for an average flight (TDATA) and the time between independent observations (DELT); then the number of independent observations on an average flight is

$$n = \frac{\text{TDATA}}{\text{DELT}}$$

- (4) Calculate the probability that the cabin ozone level exceeds MAXLMT on an average flight as

$$P(k \geq 1) = 1 - P(k = 0)$$

where

$$P(k) = \frac{n!}{k!(n-k)!} (p)^k (1-p)^{(n-k)}$$

#### Example:

- (1) For an  $M$  of 0.090 ppmv and an  $SD$  of 0.075 ppmv (all observations - from table IV),

$$\mu = \ln(0.090) - 0.5 \ln \left[ \left( \frac{0.075}{0.090} \right)^2 + 1 \right] = -2.672$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.075}{0.090} \right)^2 + 1 \right]} = 0.726$$

-- (2) For an MAXLMT of 0.3 ppmv,

$$(a) z = \frac{\ln(0.3) - (-2.672)}{(0.726)} = 2.021$$

$$(b) p = 1 - \theta(2.021) = 0.0216$$

(fig. 1, curve D, for a cabin ozone level of 0.3 ppmv).

(3) For a TDATA of 3.35 hours and a DELT of 0.5 hour,

$$n = \frac{3.35}{0.5} = 6.7$$

(4) For an n of 6.7 and a p of 0.0216,

$$P(k \geq 1) = 1 - \frac{6.7!}{0!(6.7 - 0)!} (0.0216)^0 (1 - 0.0216)^{6.7} = 0.136$$

(fig. 3, curve L, for a cabin ozone level of 0.3 ppmv).

In the preceding procedures it was assumed that cabin ozone data were available. In the following procedures, it is assumed that only ambient ozone data are available although, of course, the retention ratio (ratio of cabin ozone to ambient ozone) for the aircraft must be known or assumed (see the section DISCUSSION).

#### Procedure II-A1

Given: Maximum ambient ozone levels for a series of flights (maximum ambient ozone per-flight cfd) and the retention ratio

Solution:

- (1) Determine the maximum allowable ambient ozone level from the specified cabin ozone limit (MAXLMT) and the retention ratio  $r$  as

$$AMBLMT = \frac{(MAXLMT)}{r}$$

- (2) Then obtain the number of flights that encountered ambient ozone exceeding AMBLMT from the cfd.

Example:

- (1) For a MAXLMT of 0.3 ppmv and an  $r$  of 0.465 (from table IV),

$$AMBLMT = \frac{0.3}{0.465} = 0.645 \text{ ppmv}$$

(2) From table IV, for an AMBLMT of 0.645 ppmv,

$$P = \frac{13}{70} = 0.186$$

(fig. 4, curve M, for an ambient ozone level of 0.645 ppmv).

#### Procedure II-A2

Given: Maximum ambient ozone mean and standard deviation for a series of flights and the retention ratio

Solution:

- (1) Determine the maximum allowable ambient ozone level from the specified cabin ozone limit (MAXLMT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{MAXLMT}}{r}$$

- (2) Assume that the ambient ozone maximum-per-flight data can be approximated by an extreme-value distribution (ref. 7), and calculate the parameters  $\mu$  and  $\sigma$  for this distribution from the known mean  $M$  and standard deviation  $SD$  as

$$\mu = M - \frac{0.5776 \text{ SD}}{\sqrt{1.645}}$$

$$\sigma = \frac{\text{SD}}{\sqrt{1.645}}$$

- (3) The probability that the maximum ambient ozone level will exceed AMBLMT is given by

$$P = 1 - \exp \left[ -\exp \left( -\frac{\text{AMBLMT} - \mu}{\sigma} \right) \right]$$

Example:

- (1) For a MAXLMT of 0.3 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.3}{0.465} = 0.645 \text{ ppmv}$$

- (2) For an  $M$  of 0.415 ppmv and an  $SD$  of 0.247 ppmv (calculated from ambient ozone maximum-per-flight data in table IV),

$$\mu = 0.415 - \frac{(0.5776)(0.247)}{\sqrt{1.645}} = 0.303$$

$$\sigma = \frac{0.247}{\sqrt{1.645}} = 0.193$$

(3) Then for an AMBLMT of 0.645 ppmv,

$$P = 1 - \exp \left[ -\exp \left( -\frac{0.645 - 0.303}{0.193} \right) \right] = 0.156$$

(fig. 4, curve N, for an ambient ozone level of 0.645 ppmv).

### Procedure II-A3

Given: Ambient ozone level for a series of observations (ambient ozone cfd - all observations) and the retention ratio

Solution:

- (1) Determine the maximum allowable ambient ozone level from the specified cabin ozone limit (MAXLMT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{MAXLMT}}{r}$$

- (2) Determine the probability  $p$  that the ambient ozone level exceeds AMBLMT on any observation from the cfd.
- (3) Determine the number of independent observations per flight as

$$n = \frac{\text{TDATA}}{\text{DELT}}$$

where TDATA is the time at cruise for an average flight and DELT is the time between independent observations.

- (4) The probability of encountering exactly  $k$  observations per flight with an ambient ozone level exceeding AMBLMT is

$$P(k) = \frac{n!}{k!(n-k)!} (p)^k (1-p)^{(n-k)}$$

and the probability of encountering no observations with an ambient ozone level exceeding AMBLMT is

$$P(k \geq 1) = 1 - P(k = 0)$$

Example:

- (1) For an MAXLMT of 0.3 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.3}{0.465} = 0.645 \text{ ppmv}$$

- (2) From table V, for an AMBLMT of 0.645 ppmv

$$p = 0.0513$$

(fig. 2, curve G, for an ambient ozone level of 0.645 ppmv).

- (3) For a TDATA of 3.35 hours and a DELT of 0.5 hour,

$$n = \frac{3.35}{0.5} = 6.7$$

- (4) For an  $n$  of 6.7 and a  $p$  of 0.0513,

$$P(k \geq 1) = 1 - \frac{6.7!}{(0!)(6.7!)} (0.0513)^0 (1 - 0.0513)^{6.7} = 0.297$$

(fig. 4, curve O, for an ambient ozone level of 0.645 ppmv).

Procedure II-A4

Given: Ambient ozone mean and standard deviation for a series of observations and the retention ratio

Solution:

- (1) Determine the maximum allowable ambient ozone level from a specified cabin ozone limit (MAXLMT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{\text{MAXLMT}}{r}$$

- (2) Assume that the frequency distribution of the ambient ozone measurements is log-normal (see the section DISCUSSION), with the mean  $M$  and the standard deviation  $SD$ , and calculate the parameters  $\mu$  and  $\sigma$  for the assumed distribution as (e.g., ref. 7):

$$\mu = \ln(M) - 0.5 \ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{SD}{M} \right)^2 + 1 \right]}$$



(3) Determine the probability  $p$  that the ambient ozone level exceeds AMBLMT on any observation, from the standardized normal distribution, as follows:

(a) Calculate the number of standard deviations from the log-mean.

$$z = \frac{\ln(\text{AMBLMT}) - \mu}{\sigma}$$

(b) Obtain the probability that AMBLMT is exceeded on any observation from the cumulative density function  $\theta$  (e.g., ref. 8).

$$p = 1 - \theta(z)$$

(4) Determine the time at cruise for an average flight (TDATA) and the time between independent observations (DELTA); then the number of independent observations on an average flight is

$$n = \frac{\text{TDATA}}{\text{DELTA}}$$

(5) Calculate the probability that the ambient ozone level exceeds AMBLMT (cabin ozone > MAXLMT) on an average flight as

$$P(k \geq 1) = 1 - P(k = 0)$$

where

$$P(k) = \frac{n!}{k!(n-k)!} (p)^k (1-p)^{(n-k)}$$

#### Example:

(1) For an MAXLMT of 0.3 ppmv and an  $r$  of 0.465,

$$\text{AMBLMT} = \frac{0.3}{0.465} = 0.645 \text{ ppmv}$$

(2) For an  $M$  of 0.191 ppmv and an SD of 0.185 ppmv (all observations - from table IV),

$$\mu = \ln(0.191) - 0.5 \ln \left[ \left( \frac{0.185}{0.191} \right)^2 + 1 \right] = -1.986$$

$$\sigma = \sqrt{\ln \left[ \left( \frac{0.185}{0.191} \right)^2 + 1 \right]} = 0.813$$

(3) For an AMBLMT of 0.645 ppmv,

$$(a) z = \frac{\ln(0.645) - (-1.986)}{(0.813)} = 1.903$$

$$(b) p = 1 - \theta(1.903) = 0.0285$$

(fig. 2, curve H, for an ambient ozone level of 0.645 ppmv).

(4) For a TDATA of 3.35 hours and a DELT of 0.5 hour,

$$n = \frac{3.35}{0.5} = 6.7$$

(5) For an  $n$  of 6.7 and a  $p$  of 0.0285,

$$P(k \geq 1) = 1 - \frac{6.7!}{0!6.7!} (0.0285)^0 (1 - 0.0285)^{6.7} = 0.176$$

(fig. 4, curve P, for an ambient ozone level of 0.645 ppmv).

### SOLUTION PROCEDURES FOR PROBLEM III

For a given aircraft type and given routes (latitude, altitude, and date ranges), estimate the fraction of flights on which a specified maximum 2-hour-average cabin ozone level would be exceeded (appendix A, article 25.832 (b)).

#### Procedure III-C1

Given: Maximum 2-hour-average cabin ozone data for a series of flights (maximum 2-hr-average cabin ozone per-flight cfd)

Solution: First, the probability of encountering a 2-hour-average ozone level that exceeds a specified limit, say 2HRLMT, is (1) greater than the probability of encountering a cruise-mean level that exceeds 2HRLMT and (2) less than the probability of encountering a maximum cabin ozone level that exceeds 2HRLMT. However, since we have assumed that appropriate 2-hour-maximum data are available, the required estimate can be obtained directly.

Example: Let 2HRLMT equal 0.1 ppmv; then from table IV,

$$P = \frac{34}{70} = 0.486$$

(fig. 5, curve Q, for a cabin ozone level of 0.1 ppmv).

For comparison, the frequency of encountering a cruise-mean level exceeding 0.1 ppmv is 0.343 (curve A), and the frequency of encountering a maximum cabin ozone level exceeding 0.1 ppm is 0.829 (curve I).

### Procedure III-A1

Given: Maximum 2-hour-average ambient ozone data for a series of flights (maximum 2-hr-average ambient ozone per-flight cfd) and the retention ratio

Solution:

- (1) Determine the maximum allowable ambient ozone level from the cabin ozone limit (2HRLMT) and the retention ratio  $r$  as

$$\text{AMBLMT} = \frac{2\text{HRLMT}}{r}$$

- (2) Then obtain the fraction of flights that encountered an ambient ozone level exceeding AMBLMT from the cfd.

Example:

- (1) For a 2HRLMT of 0.1 ppmv and an  $r$  of 0.465 (table IV),

$$\text{AMBLMT} = \frac{0.1}{0.465} = 0.215 \text{ ppmv}$$

- (2) Then from table IV, for an AMBLMT of 0.215 ppmv,

$$P = \frac{31}{70} = 0.443$$

(fig. 5, curve R, for an ambient ozone level of 0.215 ppmv).

### DISCUSSION

The preceding analyses have illustrated direct solutions and several alternatives (based on limited information) for estimating the probability of exceeding specified cabin ozone limits for flight-segment-mean levels (problem I), maximum-per-flight levels (problem II), and mean levels over any 2-hour interval (problem III). The required input information and the example-case solution for each procedure are summarized in tables I to III for problems I to III, respectively.

### CRITICAL ASSUMPTIONS AND COMPARISON OF EXAMPLE-CASE SOLUTIONS

Several assumptions made in the analysis warrant further discussion. These assumptions are (1) that the probability of the cruise-mean level exceeding a specified limit equals the probability of any observation exceeding the same limit (procedures I-C3, I-C4, I-A3, and I-A4); (2) that the frequency distributions of cabin and ambient

ozone are log-normal (procedures I-C2, I-C4, I-A2, I-A4, II-C4, and II-A4); (3) that the retention ratio (ratio of cabin ozone to ambient ozone) is constant for a given aircraft (procedures I-A1, I-A2, I-A3, I-A4, II-A1, II-A2, II-A3, and II-A4); and (4) that the transformation from all-observations data to maximum-per-flight data can be made by using the average number of independent observations per flight determined from the characteristic scale of ambient ozone variability (procedures II-C3, II-C4, II-A3, and II-A4).

### Cruise Mean Level

It is apparent from the good agreement between the several procedures (figs. 1 and 2 and table I) that the cumulative frequency distribution for all observations provides a reasonable estimate for the cruise-mean per-flight cumulative frequency distribution. Although curves A and C, and E and G, are based on exactly the same set of measurements, they are averaged (weighted) differently; so we would not expect them to be identical. Specifically, curves C and G are based on flight averages, and their standard deviation from the mean should be less than the standard deviation of all observations from the mean. Therefore the slope of these curves near the mean is greater than the slope of the all-observations curves, as shown in figures 1 and 2. At low cabin and ambient ozone levels, the probability that the cruise mean exceeds a specified limit is greater than the probability that any observation exceeds that limit. At higher cabin and ambient ozone levels, the probability that the cruise mean exceeds a specified limit is less than the probability that any observation exceeds the same limit.

### Log-Normal Distribution

In procedures for which the known ozone data are limited to mean and standard-deviation values, the frequency distribution of the observations with respect to ozone level must be assumed. First, we know that this distribution cannot be normal (Gaussian) about the mean (1) since the most frequently occurring value (mode) is less than the average (mean) and (2) since ozone levels must equal or exceed 0. Thus we cannot infer probabilities directly from the mean, the mean plus 1 standard deviation, etc.

An alternative assumption, used herein, is that the logarithms of the ozone level are normally distributed about the log-mean. This log-normal distribution has been used to satisfactorily model many physical processes, including pollutant distributions (refs. 10 and 11). The frequency distribution of the cabin and ambient ozone data in the example case and the corresponding log-normal distribution calculated from the mean and standard-deviation values are shown in figure 6.

The next important question is, Can we expect all distributions to be log-normal? Unfortunately, the answer appears to be no, since the frequency distribution for any given aircraft will reflect its route structure and cruise altitudes. As an example, ambient ozone frequency distributions for three Boeing 747's in the second quarter (April to June) of 1975 and 1976 are shown in figure 7. The distribution for the 747-100 in domestic service is very similar to the distribution shown in figure 6(b). However, the distribution for the 747-100 in international service shows a much larger fraction of data in the 0- to 0.005-ppmv interval as a consequence of flights in the tropics. The distribution for the 747SP is clearly bimodal, with the first mode resulting from a few short flights at low altitudes and ascent and descent data, and the second mode resulting from high-altitude flights at northerly latitudes, where the aircraft was well into the stratosphere. Thus, the log-normal assumption would seem reasonable only for data in which the geographical region and altitude range of the observations are suitably restricted.

#### Retention Ratio

In the analyses and example cases, we have assumed that the retention ratio (ratio of cabin ozone to ambient ozone) was constant for a particular aircraft type. In figure 8, the mean retention ratios for 0.05-ppmv intervals in ambient ozone suggest that this ratio decreases with increasing ambient ozone level. This is exactly what would be expected if the dominant mechanism destroying ozone in the cabin air were thermal (ref. 12), as would be the case in the engine compressor.

In light of figure 8, it is instructive to reexamine the results shown in figures 1 to 5. The abscissa scale for ambient ozone in figures 2 and 4 was chosen to provide a direct comparison with the cabin ozone estimates in figures 1 and 3 for a constant retention ratio (0.465), and the abscissa in figure 5 is double labeled in the same manner. A careful comparison of figures 1 and 2 does suggest that the curves in figure 2, particularly curves G and H, should be shifted slightly to the left at high cabin ozone levels (smaller retention ratios) and should be shifted slightly to the right at low cabin ozone levels (larger retention ratios), although the correction is not large. The same is true for curves O and P in figure 4.

Also relevant to this comparison are the curves in figure 5 for the maximum, 2-hour-maximum, and cruise-mean limits. These curves all represent direct solutions - the only difference being that, for each pair of curves, one is based on cabin ozone data and the other is based on ambient ozone data. Here we see that the good agreement between the cabin-based and ambient-based probabilities for all problems would be substantially worsened if the ambient curves were adjusted for a varying retention ratio.



These comparisons suggest that, although the relationship between the retention ratio and ambient ozone level shown in figure 8 appears to be unambiguous, other factors may be important also. Perhaps, as suggested in reference 5, the retention ratio varies with flight duration and load factor; or perhaps the exchange rate of cabin air, which causes a time lag between the "simultaneous" cabin and ambient measurements, has an important effect on maximum-value or short-time averages but has little or no effect on all-observations results. Because of these uncertainties, we have elected not to introduce the complexity of a varying retention ratio into the calculation procedures.

### Maximum Values

In problem II, except for the procedures where maximum-per-flight data are given, the probability that a specified limit will be exceeded on a given flight must be obtained from the known probability that the limit will be exceeded on any observation. This, at first, appears to be straightforward because the average flight duration for any given aircraft or routes can easily be obtained and because there are eight GASP ozone observations each hour (ref. 1). However, because of the characteristic scale of ambient ozone variability, not all of these observations are independent.

In reference 9, from an analysis of the time-lagged autocorrelation function from 33 carefully selected GASP flights (constant-level, turbulence-free flight for at least 1200 km, with no tropopause crossings), the east-west distance between independent observations  $2d$  was found to be 450 kilometers (or 30 min). However, as pointed out therein, most GASP flights do not fulfill these criteria. Thus, this estimate does not necessarily represent all GASP data. In particular, altitude changes, turbulence, and tropopause crossings would all act to increase the number of independent observations. In the absence of specific data as to the magnitude of these effects, we have elected to use the 30-minute interval in these calculations.

With this estimate and the average flight duration, we can define an equivalent number of independent observations per flight. This, along with the probability that a specified limit will be exceeded on any observation, provides all the information necessary to estimate the probability that any number of observations will exceed the limit, by using the binomial formula (e.g., ref. 8).

The resulting maximum-per-flight cumulative frequency distributions for the example cases are shown in figures 3 and 4. It is immediately apparent that the envelope of these solutions is substantially larger than the envelope of the solutions to problem I (figs. 1 and 2). This results from the exponentiation in the binomial formula; for example, an uncertainty of  $\pm 0.02$  at a probability of 0.05 that any observation will exceed a limit will result in an uncertainty of  $\pm 0.1$  that the limit will be exceeded on the aver-

age flight (where the number of observations  $n$  is 6.7). Also, an uncertainty of  $\pm 1$  in the number of independent observations per flight will, at the same probability of 0.05 that any observation will exceed the limit, result in an uncertainty of  $\pm 0.04$  that the limit will be exceeded on an average flight. Because of these factors, the envelope of solutions to problem II is approximately  $\pm 0.1$  from the direct solution, compared with an envelope of approximately  $\pm 0.05$  for solutions to problem I.

In solving problem II, the maximum-value-per-flight data require the fewest number of assumptions, but this approach reduces the data base used to one observation per flight. Thus, the statistical confidence level for the maximum-value cfd's is less than that for the all-observation cfd's.

### CALCULATIONS USING TABULATED AMBIENT OZONE DATA

From the example-case solutions in tables I and II, it is apparent that procedures I-A4 and II-A4 (which are based on ambient ozone mean and standard-deviation values) are in excellent agreement with the direct solution for each problem. This suggests that these procedures would be extremely useful in estimating the occurrence of high cabin ozone encounters from tabulated ambient ozone data available in the literature. One such tabulation is given in reference 3, where appendix A data are from ozone-sondes and appendix B data are from GASP observations for 1975-76.

Examination of the flight data in table IV reveals (1) that most of the flights were in April; (2) that the mean flight level was 369 (11.2 km; 36 900 ft); and (3) that the flights were geographically located between  $21^{\circ}$  and  $48^{\circ}$  N latitude and  $74^{\circ}$  and  $150^{\circ}$  W longitude, with all flights either arriving or departing (or both) in the Western United States. Thus, a reasonable approximation to these flights is provided by the data in reference 3 (appendix A) for April, Western North America, and flight level 370. The latitudes of the flights suggest that the tabulated values for  $35^{\circ}$  N and  $40^{\circ}$  N should be averaged. For this case we obtain a mean  $M$  of 0.195 ppmv and a standard deviation  $SD$  of 0.171 ppmv. (The tabulations headed "16%" in ref. 3 are mean-plus-1-standard-deviation values.)

Thus, by reworking the example-case calculations in procedure I-A4 for an  $M$  of 0.195 ppmv and an  $SD$  of 0.171 ppmv, we estimate the probability  $P$  of the flight-segment-average cabin ozone level exceeding the specified limit of 0.1 ppmv to be 0.245. The probability for the direct solution (procedure I-C1) is 0.243. Similarly, by using these mean and standard-deviation values in procedure II-A4, we estimate the probability  $P$  of encountering a maximum value greater than the specified 0.3-ppmv limit to be 0.156. The direct solution (procedure II-C1) yields a probability of 0.200.

Another source of ambient ozone mean and standard-deviation values is the GASP data tabulated in appendix B of reference 3. Here, the best approximation to the flights in table IV is provided by the tabulation for March, April, and May: 37 000 feet altitude (flight level 370);  $90^{\circ}$  to  $140^{\circ}$  W longitude; and  $24^{\circ}$  to  $42^{\circ}$  N latitude. From this table we obtain a mean  $M$  of 0.199 ppmv and a standard deviation  $SD$  of 0.170 ppmv.

By reworking the example-case calculations in procedure I-A4 for an  $M$  of 0.199 ppmv and an  $SD$  of 0.170 ppmv, we estimate the probability that the flight-segment-average cabin ozone level exceeds 0.1 ppmv to be 0.254. The direct solution yields 0.243, and procedure I-A4 with the data of reference 3 (appendix A) yields 0.245. Similarly, for an  $M$  of 0.199 ppmv and an  $SD$  of 0.170 ppmv in procedure II-A4, the probability of encountering a maximum value greater than 0.3 ppmv is 0.156. The direct solution (procedure II-C1) yields 0.200, and procedure II-A4 using the data of reference 3 (appendix A) yields 0.156. For both problems I and II, the estimates made from the ambient ozone data tabulated in reference 3 are within the envelope of the several example-case solutions (tables I and II).

It was mentioned in the previous section that the assumption of a log-normal frequency distribution for the ozone measurements was a critical step in several procedures. To emphasize this, consider the following calculations in which the input data from the previous paragraph are used, but with the ozone frequency distribution assumed to be "normal" (Gaussian) rather than log-normal.

For procedure I-A4, let the specified cabin ozone flight-segment-average limit be 0.1 ppmv, let  $M$  be 0.199 ppmv, and let  $SD$  be 0.170 ppmv. Then for a Gaussian distribution,  $z = (0.247 - 0.199)/0.170 = 0.282$  and the probability of the flight-segment-average cabin ozone level exceeding the specified limit is estimated to be 0.389, which is 60 percent too high. Similarly, in procedure II-A4, for a specified maximum cabin ozone limit of 0.3 ppmv and a Gaussian distribution,  $z = (0.645 - 0.199)/0.170 = 2.623$  and the probability of the maximum cabin ozone level exceeding the specified limit is estimated to be 0.029, which is 86 percent too low.

Thus, the results presented herein show that the assumed form of the frequency distribution is indeed critical and that gross errors can result if the distribution is assumed to be Gaussian when it is not. The log-normal assumption would be expected to be valid for tabulated data since these are restricted in altitude, season, and geographical region and since the widely varying altitudes and routes that can cause the aircraft distributions to be other than log-normal (fig. 7) are not present in tabulated data.

## EFFECT ON PROBABILITY ESTIMATES OF VARYING LATITUDE, ALTITUDE, SEASON, RETENTION RATIO, FLIGHT DURATION, AND CABIN OZONE LIMITS

The success with which the direct example-case solutions are approximated by using the reference 3 ambient ozone data suggests using these data to evaluate how variations in each of the independent variables in the solution affect the estimated probabilities. These parameters are the latitude, altitude, and season (affecting ambient zone mean and standard-deviation levels); the retention ratio (ratio of cabin ozone to ambient ozone), which depends on the aircraft type; and the number of independent observations per flight, which varies with flight duration. (This last parameter is only required for problem II solutions.) Also examined is the effect of varying the specified cabin ozone limits on the estimated probabilities.

Figure 9 shows the results obtained for problem I (solid curves) and problem II (dashed curves) by using flight data from table IV, ambient ozone data from appendix B of reference 3, and cabin ozone limits from reference 6. In each part of this figure, one parameter is allowed to vary over its range of possible values, while the others are held constant. The symbols identify the data point, common to all parts of the figure, that most nearly corresponds to the example-case conditions. Specific results in parts (a) to (f), respectively, of figure 9 are as follows:

(1) Figure 9(a): The probability of exceeding cabin ozone limits increases markedly at northerly latitudes, as would be expected since the mean ambient ozone level increases sharply with increasing latitude. The dip in the curves for  $48^{\circ}$  to  $54^{\circ}$  N follows from a similar feature seen in both ozonesonde and GASP mean ambient ozone levels (ref. 2).

(2) Figure 9(b): The probability of exceeding specified cabin ozone limits clearly increases with increasing altitude above flight level 370 and below flight level 330. In between these altitudes, for this season and latitude, differences in mean flight level do not have much impact on cabin ozone encounter probabilities.

(3) Figure 9(c): The highest probability of exceeding cabin ozone limits occurs in the spring. This corresponds to the spring peak in ambient ozone (ref. 2) and to the greater probability of encountering high ambient ozone levels in the spring than in the other seasons (ref. 4).

(4) Figure 9(d): Of course, the likelihood of encountering high ozone levels in the cabin is greater if the retention ratio  $r$  (ratio of cabin ozone to ambient ozone) is higher. Identified on this curve, for comparison with the 747-100 results ( $r = 0.465$ ), are retention ratios for several 747SP air-conditioning configurations (ref. 1) as follows: (1) unmodified,  $r = 0.82$ ; (2) increased air recirculation,  $r = 0.55$ ; (3) high-temperature compressor bleed,  $r = 0.22$ ; and (4) charcoal filter,  $r = 0.06$ .



(5) Figure 9(e): The probability of encountering a maximum cabin ozone level in excess of the limit increases with the number of independent observations per flight. This variation can be used to examine the effect of changes in flight duration or the effect of varying the estimated time between independent observations, or both. (Note the triple-labeled abscissa in fig. 9(e).)

(6) Figure 9(f): Finally, the probability of exceeding a specified cabin ozone limit decreases as the limit increases. The symbols here, as in parts (a) to (e), denote calculated probabilities for the limits specified in reference 6.

Results such as those shown in figure 9 depend directly on the availability of statistically representative ambient ozone data. As more data become available, these results may vary slightly, and it may be possible to make reliable estimates in regions and at altitudes for which the available data are currently too limited. Also, as mentioned previously, estimates based on ambient ozone data require that the ratio of cabin ozone to ambient ozone be known, and this parameter is different for various cabin air-conditioning systems. Thus, the results in figure 9 should be interpreted as examples showing relative differences in estimated probabilities due to changes in the independent variables, but they should not be extrapolated to conditions other than those shown. For different aircraft, routes, seasons, or limits, the appropriate calculations should be performed directly.

### SUMMARY OF RESULTS

This report presents and discusses several procedures for estimating the frequency at which commercial airliner flights will encounter high cabin ozone levels. This work was motivated by rules proposed by the Federal Aviation Administration (FAA) which, if adopted, would establish limits on ozone levels in commercial airliner cabins and would require air carriers to demonstrate, by analysis or tests, that their aircraft would be in compliance with the rules.

Based on the proposed rules, three analytical problems have been posed and several solution procedures are presented for each. These procedures are addressed to different levels of input information - from complete cabin ozone measurements, which provide the required estimate directly, to limited ozone information, such as ambient ozone means and standard deviations, for which several assumptions are necessary to obtain the required estimate.

Each procedure is illustrated by an example that uses a set of simultaneous cabin and ambient ozone measurements obtained by the Global Atmospheric Sampling Program (GASP) from a Boeing 747-100 airliner in domestic U.S. service from March 30, 1977, to June 13, 1977. Using these data also permits a direct comparison of the several procedures for each problem.

The first problem solved is that of determining the fraction of flights on which a specified time-weighted, flight-segment-average cabin ozone level would be exceeded. All procedures, including that for which the known information is limited to the ambient ozone means and standard deviations and the retention ratio (ratio of cabin ozone to ambient ozone), provide estimates within  $\pm 0.05$  from the direct (experimental) solution for all cabin ozone limits greater than 0.1 ppmv.

The second problem solved is that of determining the fraction of flights on which a specified maximum cabin ozone limit would be exceeded. Solutions to this problem range from the direct solution, which uses maximum cabin ozone per-flight data, to an estimate based only on the retention ratio and the mean and standard deviation from a series of ambient ozone observations. The envelope of solutions for this problem is within  $\pm 0.1$  from the direct solution for any specified cabin ozone limit.

The final problem solved is that of determining the fraction of flights on which the maximum cabin ozone level during any 2-hour interval would exceed a specified limit. Solutions to this problem, which assume that either 2-hour-maximum cabin or 2-hour-maximum ambient ozone data are available, agree within  $\pm 0.05$  for all cabin ozone limits.

To estimate flight-segment-mean and maximum-per-flight levels, calculations are performed by using tabulated ambient ozone data from the literature. These solutions are in excellent agreement with the example-case solutions. Additional example calculations using the tabulated data show that the probability of exceeding specified cabin ozone limits increases with increasing latitude, altitude, flight duration, and retention ratio. These calculations also show maximum probabilities in the late winter and early spring; and, of course, for all conditions the probabilities decrease if the limits are increased.

The several alternative solution procedures for each problem given in this report lead to the obvious question, Which should be used? The answer to this is clearly that one should use the procedure that gives the most direct solution based on the available data. Although the assumptions made in the several alternative procedures appear reasonable, it would be unwise to discard direct experimental data in favor of a procedure requiring assumptions. When, however, the available data are limited, several of the procedures given here provide an alternative to a costly comprehensive measurements program.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, July 9, 1979,  
198-10.

## APPENDIX A

### PROPOSED RULES<sup>1</sup>

"Based on a review of comments received in response to [ref. 13], the FAA has concluded that specific airplane cabin ozone concentration standards should be established in parts 25 and 121 of the Federal Aviation regulations (14 CFR Parts 25 and 121)."

The specific terms of the proposed amendments are

(1) For new transport-category airplanes:

"Article 25.832 Cabin ozone concentration.

The airplane cabin ozone concentration during flight above flight level 180 must be shown not to exceed -

(a) 0.3 parts per million by volume, and

(b) 0.1 parts per million by volume time-weighted average during any 2-hour interval."

(2) For currently certified airplanes:

"Article 121.578 Transport category airplane: Cabin ozone concentration.

After . . . no certificate holder may operate a transport category airplane above flight level 180 unless it has successfully demonstrated to the Administrator that the concentration of ozone inside the cabin will not exceed -

(a) 0.3 parts per million by volume, and

(b) For each flight segment that exceeds 3 hours, 0.1 parts per million by volume time-weighted average over that flight segment.

For the purpose of this section "flight segment" means the scheduled non-stop flight time between any two airports."

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<sup>1</sup>From ref. 6.

## APPENDIX B

### EXAMPLE-CASE DATA SET SUMMARY

The parameters presented in table IV were selected according to the requirements of reference 6 and include, for each flight, the departure date, route, mean flight level, data time interval, and cabin and ambient ozone data. All ozone levels are expressed as mixing ratios in parts per million by volume (ppmv).

Entries appear for all flights from GASP tape VL0012 for which simultaneous cabin and ambient ozone measurements are available. The number of these observations for each flight is given in the column after the flight information. The elapsed time from the first to the last ozone observation for each flight is given in the TDATA column. Since the GASP systems do not obtain data at altitudes below 6 kilometers (flight level 195), the TDATA times here are less than the segment times specified in reference 6. The times during which the cabin ozone level exceeded 0.1 and 0.3 ppmv appear in the columns after the TDATA column.

Next, the time-weighted mean and standard-deviation, maximum, and 2-hour-maximum cabin ozone levels are given for each flight. The mean M and standard-deviation SD cabin ozone levels are averages weighted over the time given by TDATA.

Then, the time-weighted mean and standard-deviation, maximum, and 2-hour-maximum ambient ozone levels are given for each flight. For measurement periods in which the ambient ozone level is greater than 0.1 ppmv, the ratio of cabin ozone to ambient ozone (retention ratio) is calculated. The number of these data points, as well as the mean and the standard deviation, are given after the ambient ozone data.

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TABLE I. - COMPARISON OF PROCEDURES FOR ESTIMATING FRACTION OF FLIGHTS ON WHICH SPECIFIED  
FLIGHT-SEGMENT-AVERAGE CABIN OZONE LIMIT WOULD BE EXCEEDED - PROBLEM 1

Known or assumed	Data source for example cases	Calculation procedure <sup>a</sup>							
		I-C1	I-C2	I-C3	I-C4	I-A1	I-A2	I-A3	I-A4
Cruise-mean ozone per-flight cumulative frequency distribution (cfd)	Curves A and E (figs. 1 and 2); data from table IV	C				A			
Cruise-mean ozone per-flight mean and standard deviation	Curves B and F (figs. 1 and 2); data from table IV		C				A		
All-observations ozone cfd	Curves C and G (figs. 1 and 2); data from table V)			C				A	
All-observations ozone mean and standard deviation	Curves D and H (figs. 1 and 2); data from table IV				C				A
Frequency distribution	Log-normal		L		L		L		L
Retention ratio (ratio of cabin ozone to ambient ozone), r	r = 0.465 (table IV)					r	r	r	r
Example-case solutions:									
Estimated fraction of flights on which flight-segment-average cabin ozone limit of 0.1 ppmv would be exceeded		0.243	0.267	0.230	0.242	0.243	0.266	0.220	0.236
Figure		1, 5	1	1	1	2, 5	2	2	2

<sup>a</sup>For each procedure, symbols in table indicate information in left column that must be known or assumed: C = cabin ozone; A = ambient ozone; L = log-normal frequency distribution; r = retention ratio.



TABLE II. - COMPARISON OF PROCEDURES FOR ESTIMATING FRACTION OF FLIGHTS ON WHICH SPECIFIED  
MAXIMUM CABIN OZONE LIMIT WOULD BE EXCEEDED - PROBLEM II

Known or assumed	Data source for example cases	Calculation procedure <sup>a</sup>							
		II-C1	II-C2	II-C3	II-C4	II-A1	II-A2	II-A3	II-A4
Maximum ozone per-flight cumulative frequency distribution (cfd)	Curves I and M (figs. 3 and 4); data from table IV	C				A			
Maximum ozone per-flight mean and standard deviation	Curves J and N (figs. 3 and 4); data from table IV		C				A		
All-observations ozone cfd	Curves C and G (figs. 1 and 2); data from table V			C				A	
All-observations ozone mean and standard deviation	Curves D and H (figs. 1 and 2); data from table IV				C				A
Frequency distribution	Extreme-value; log-normal		E		L		E		L
Number of independent observations per flight, n	n = 6.7 (see text)			n	n			n	n
Retention ratio (ratio of cabin ozone to ambient ozone), r	r = 0.465 (table IV)					r	r	r	r
Example-case solutions:									
Estimated fraction of flights on which maximum cabin ozone limit of 0.3 ppmv would be exceeded		0.200	0.152	0.185	0.136	0.186	0.156	0.297	0.176
Figure		3, 5	3	3	3	4, 5	4	4	4

<sup>a</sup>For each procedure, symbols in table indicate information in left column that must be known or assumed: C = cabin ozone; A = ambient ozone; E = extreme-value frequency distribution; L = log-normal frequency distribution; n = number of independent observations per flight; r = retention ratio.



TABLE III. - COMPARISON OF PROCEDURES FOR ESTIMATING FRACTION OF  
 FLIGHTS ON WHICH SPECIFIED 2-HOUR-MAXIMUM CABIN OZONE LIMIT  
 WOULD BE EXCEEDED - PROBLEM III

Known or assumed	Data source for example cases	Calculation procedure <sup>a</sup>	
		III-C1	III-A1
2-Hour-maximum ozone per-flight cumulative frequency distribution (cfd) Retention ratio (ratio of cabin ozone to ambient ozone), r	Curves Q and R (fig. 5); data from table IV  r = 0.465 (table V)	C	A  r
Example-case solutions: Estimated fraction of flights on which 2-hour-maximum cabin ozone limit of 0.1 ppmv would be exceeded Figure		0.486  5	0.443  5

<sup>a</sup>For each procedure, symbols in table indicate information in left column which must be known or assumed: C = cabin ozone; A = ambient ozone; r = retention ratio.

46

TABLE IV. - EXAMPLE-CASE DATA SET SUMMARY

[Boeing 747-100; GASP tape VL0012, file 2.]

Date	Route <sup>a</sup>	Flight level <sup>b</sup>	Number of observations	Time at cruise, TDATA, hr:min	Times, hr:min, during which cabin ozone level exceeded -		Cabin ozone level, ppmv				Ambient ozone level, ppmv				Retention ratio (ratio of cabin ozone to ambient ozone), r, for ambient >0.1 ppmv		
					0.1 ppmv	0.3 ppmv	Mean, M	Standard deviation, SD	Maximum	2-Hour maximum	Mean, M	Standard deviation, SD	Maximum	2-Hour maximum	Number of observations	Mean, M	Standard deviation
3/30/77	LAX-JFK	372	28	3:28	:00	:00	0.038	0.018	0.092	0.045	0.091	0.023	0.131	0.105	12	0.378	0.129
3/31/77	JFK-LAX	370	42	5:11	3:31	:45	.159	.096	.377	.221	.335	.245	.854	.492	29	.508	.233
4/1/77	LAX-SEA	385	10	1:18	1:18	:13	.275	.041	.323	.275	.691	.051	.765	.691	10	.389	.063
4/1/77	SEA-ORD	357	17	2:25	2:15	1:25	.288	.092	.395	.324	.605	.236	.851	.686	15	.471	.143
4/1/77	ORD-LAX	390	25	3:10	2:45	1:20	.246	.103	.375	.305	.644	.279	.991	.831	24	.374	.069
4/2/77	LAX-JFK	357	29	3:46	2:40	:00	.171	.078	.296	.234	.421	.288	.807	.647	26	.472	.253
4/3/77	JFK-LAX	377	34	4:41	4:16	:55	.216	.096	.411	.302	.605	.271	1.044	.879	33	.363	.067
4/4/77	LAX-HNL	358	31	4:02	:15	:00	.052	.026	.145	.063	.091	.024	.151	.101	14	.459	.156
4/4/77	HNL-SFO	361	23	3:40	:00	:00	.048	.021	.099	.053	.104	.040	.223	.114	11	.365	.132
4/5/77	HNL-SFO	358	29	3:50	:00	:00	.054	.019	.095	.058	.122	.024	.171	.127	24	.438	.147
4/6/77	SFO-ORD	371	22	2:41	:25	:00	.077	.037	.171	.074	.181	.094	.386	.196	16	.474	.295
4/7/77	ORD-LAX	382	22	2:45	:55	:10	.106	.085	.377	.126	.143	.056	.238	.168	17	.544	.319
4/7/77	LAX-JFK	385	28	3:45	2:25	:30	.145	.091	.369	.205	.320	.173	.678	.471	28	.473	.167
4/8/77	JFK-LAX	373	32	4:25	2:25	:30	.139	.094	.322	.221	.319	.211	.746	.495	29	.438	.116
4/9/77	LAX-SEA	388	9	1:00	:50	:00	.132	.060	.237	.132	.408	.219	.834	.408	9	.358	.128
4/9/77	SEA-ORD	389	20	2:34	1:15	:30	.154	.109	.389	.169	.287	.196	.723	.324	18	.508	.172
4/9/77	ORD-LAX	385	22	2:50	:34	:00	.076	.030	.126	.087	.159	.067	.299	.176	17	.462	.131
4/10/77	LAX-HNL	394	30	4:25	1:55	:00	.097	.061	.217	.133	.188	.125	.561	.196	21	.557	.266
4/10/77	HNL-SFO	359	17	3:42	:00	:00	.052	.022	.089	.054	.127	.047	.224	.144	9	.347	.105
4/11/77	SFO-HNL	399	27	4:05	1:10	:00	.082	.035	.154	.086	.163	.077	.330	.175	22	.496	.158
4/11/77	HNL-SFO	368	27	3:45	:10	:00	.043	.027	.127	.060	.100	.051	.239	.110	7	.428	.324
4/11/77	SFO-ORD	367	23	2:52	1:57	:00	.147	.069	.266	.173	.321	.135	.539	.375	21	.451	.109
4/12/77	ORD-LAX	397	25	2:55	2:05	:25	.181	.098	.372	.238	.447	.276	.868	.613	22	.454	.170
4/12/77	LAX-JFK	369	31	3:54	1:05	:00	.078	.055	.237	.106	.160	.114	.544	.221	17	.459	.130
4/13/77	JFK-LAX	372	33	4:20	:30	:00	.067	.030	.163	.085	.148	.097	.471	.209	21	.430	.104
4/14/77	LAX-HNL	357	32	4:06	:10	:00	.060	.026	.143	.069	.081	.019	.120	.092	7	.627	.337
4/14/77	HNL-SFO	366	32	3:55	:15	:00	.045	.024	.132	.049	.090	.022	.133	.106	12	.464	.233
4/15/77	SFO-HNL	343	31	4:01	:00	:00	.036	.010	.063	.038	.079	.020	.112	.096	5	.343	.051
4/15/77	HNL-SFO	369	30	3:36	:04	:00	.043	.017	.112	.046	.090	.013	.118	.095	7	.462	.180
4/15/77	SFO-ORD	367	22	2:50	:55	:00	.090	.042	.192	.102	.184	.124	.639	.223	14	.465	.235
4/16/77	ORD-LAX	390	12	1:30	:45	:00	.091	.023	.131	.091	.164	.084	.370	.164	9	.586	.214
4/16/77	LAX-HNL	350	33	4:15	:00	:00	.034	.014	.078	.036	.076	.016	.105	.084	1	.333	.00
4/17/77	HNL-SFO	364	14	1:40	:00	:00	.041	.017	.076	.041	.082	.013	.096	.082	0	.00	.00
4/19/77	SFO-HNL	355	36	4:25	:45	:00	.075	.063	.264	.111	.144	.129	.528	.230	11	.461	.164
4/20/77	HNL-LAX	373	28	3:35	:10	:00	.055	.026	.127	.064	.098	.052	.228	.121	11	.501	.149

4/20/77	LAX-JFK	369	29	3:50	1:15	:00	.087	.052	.241	.122	.193	.141	.588	.298	19	.481	.176
4/21/77	JFK-LAX	379	33	4:06	1:35	:00	.106	.056	.251	.140	.202	.205	.683	.358	16	.444	.283
4/22/77	LAX-HNL	353	27	4:24	:20	:00	.060	.025	.148	.065	.099	.049	.343	.112	9	.463	.118
4/22/77	HNL-LAS	359	35	4:20	:00	:00	.030	.018	.090	.044	.079	.042	.208	.109	9	.370	.088
4/22/77	LAS-LAX	219	1	:00	:00	:00	.017	.00	.017	.017	.046	.00	.046	.046	0	.00	.00
4/22/77	LAX-JFK	368	30	3:50	:20	:00	.053	.038	.219	.071	.128	.113	.465	.191	14	.380	.152
4/23/77	JFK-LAX	375	21	4:15	1:45	:00	.088	.040	.182	.108	.215	.132	.495	.287	18	.396	.253
4/24/77	LAX-HNL	359	32	4:16	1:01	:00	.062	.037	.169	.087	.136	.117	.396	.235	10	.299	.116
4/24/77	HNL-LAX	335	27	3:45	:25	:00	.047	.032	.127	.064	.100	.078	.298	.142	6	.365	.047
4/24/77	LAX-ORD	354	8	2:40	1:10	:00	.102	.067	.202	.129	.301	.295	.750	.390	4	.395	.156
4/24/77	ORD-LAX	387	5	1:15	1:11	:00	.139	.044	.191	.139	.265	.052	.313	.265	5	.480	.099
4/25/77	LAX-HNL	356	37	4:45	:10	:00	.046	.031	.182	.054	.070	.020	.110	.080	2	.395	.014
4/26/77	HNL-SFO	364	27	3:25	:39	:00	.064	.030	.126	.081	.085	.050	.199	.120	9	.630	.202
4/26/77	SFO-HNL	350	28	4:30	:00	:00	.055	.017	.098	.062	.114	.036	.187	.127	14	.482	.085
4/26/77	HNL-LAX	372	28	3:40	1:35	:10	.108	.073	.458	.138	.212	.058	.308	.260	26	.520	.317
4/27/77	LAX-HNL	357	28	4:35	:00	:00	.054	.013	.072	.063	.109	.023	.164	.128	15	.439	.092
4/27/77	HNL-SFO	368	27	3:23	:44	:00	.073	.046	.193	.094	.147	.092	.365	.193	12	.489	.137
4/28/77	SFO-ORD	354	21	2:40	:10	:00	.061	.021	.106	.068	.116	.052	.278	.129	12	.510	.105
4/29/77	ORD-SEA	391	24	3:00	2:00	:10	.128	.069	.320	.145	.284	.129	.575	.313	23	.449	.115
4/29/77	SEA-ORD	384	18	2:20	1:20	:00	.101	.032	.151	.101	.232	.090	.419	.238	16	.456	.153
4/29/77	ORD-SFO	381	22	3:10	:15	:00	.065	.034	.141	.084	.141	.062	.250	.174	13	.457	.144
4/30/77	SFO-JFK	386	29	3:55	2:05	:00	.118	.076	.293	.169	.234	.147	.496	.340	20	.542	.324
5/1/77	JFK-SFO	364	34	4:25	1:30	:05	.102	.081	.316	.178	.218	.192	.622	.408	17	.450	.148
5/1/77	SFO-HNL	373	25	3:50	1:40	:00	.091	.054	.190	.133	.165	.111	.416	.234	13	.531	.159
5/2/77	HNL-SFO	363	32	4:10	:15	:00	.048	.017	.109	.054	.089	.033	.176	.105	5	.417	.102
5/3/77	SFO-ORD	349	17	2:45	:40	:00	.086	.047	.211	.107	.183	.099	.446	.228	13	.487	.138
5/4/77	ORD-SEA	391	14	2:36	2:36	:21	.199	.069	.308	.230	.401	.118	.631	.451	14	.530	.259
5/4/77	ORD-LAX	401	22	3:01	:53	:00	.096	.067	.286	.110	.161	.124	.520	.201	13	.604	.226
5/31/77	SFO-HNL	388	28	3:55	:00	:00	.030	.013	.052	.036	.067	.019	.107	.077	1	.290	.00
6/10/77	JFK-LAX	388	20	3:04	:50	:00	.092	.038	.194	.105	.134	.080	.311	.170	11	.706	.318
6/10/77	LAX-HNL	358	20	4:09	:14	:00	.054	.024	.110	.064	.066	.027	.116	.080	2	.412	.028
6/11/77	LAX-ORD	361	18	2:30	:40	:00	.085	.035	.173	.091	.154	.099	.396	.179	13	.542	.322
6/12/77	ORD-SEA	400	7	:55	:55	:00	.226	.023	.256	.226	.447	.062	.508	.447	7	.536	.113
6/12/77	SEA-ORD	368	7	2:10	:05	:00	.065	.029	.106	.071	.139	.073	.241	.157	4	.387	.128
6/13/77	HNL-SFO	351	10	1:20	:40	:00	.112	.064	.267	.112	.227	.144	.481	.227	7	.451	.102
All observations		369	1697	234:16	66:43	7:29	0.090	0.075	0.458	0.324	0.191	0.185	1.044	0.879	941	0.465	0.201

<sup>a</sup> LAX denotes Los Angeles; JFK denotes New York; SEA denotes Seattle; ORD denotes Chicago; HNL denotes Honolulu; SFO denotes San Francisco, LAS denotes Las Vegas.

<sup>b</sup> Flight level refers to altitude in hundreds of feet.

41

TABLE V. - ALL-OBSERVATIONS CUMULATIVE FREQUENCY DISTRIBUTIONS  
FOR EXAMPLE-CASE DATA SET

Cabin ozone			Ambient ozone		
Limit, ppmv	Number of observations, n	Fraction of time limit is exceeded	Limit, ppmv	Number of observations, n	Fraction of time limit is exceeded
$\geq 0.0$	1697	1.000	$\geq 0.0$	1697	1.000
$\geq .025$	1570	.925	$\geq .05$	1591	.937
$\geq .05$	1063	.626	$\geq .1$	941	.555
$\geq .075$	679	.400	$\geq .15$	624	.368
$\geq .1$	475	.280	$\geq .2$	478	.282
$\geq .115$	391	.230	$\geq .247$	374	.220
$\geq .125$	356	.210	$\geq .25$	372	.219
$\geq .15$	266	.157	$\geq .3$	302	.178
$\geq .175$	228	.134	$\geq .35$	255	.150
$\geq .2$	181	.107	$\geq .4$	213	.123
$\geq .225$	143	.084	$\geq .45$	181	.107
$\geq .25$	107	.063	$\geq .5$	157	.093
$\geq .275$	73	.043	$\geq .55$	127	.075
$\geq .3$	51	.030	$\geq .6$	102	.060
$\geq .325$	32	.019	$\geq .645$	87	.051
$\geq .35$	17	.010	$\geq .65$	84	.049
$\geq .375$	8	.0047	$\geq .7$	60	.035
$\geq .4$	2	.0012	$\geq .75$	48	.028
$\geq .425$	1	.0006	$\geq .8$	27	.016
$\geq .45$	1	.0006	$\geq .85$	19	.011
			$\geq .9$	8	.0047
			$\geq .95$	6	.0035
			$\geq 1$	2	.0012

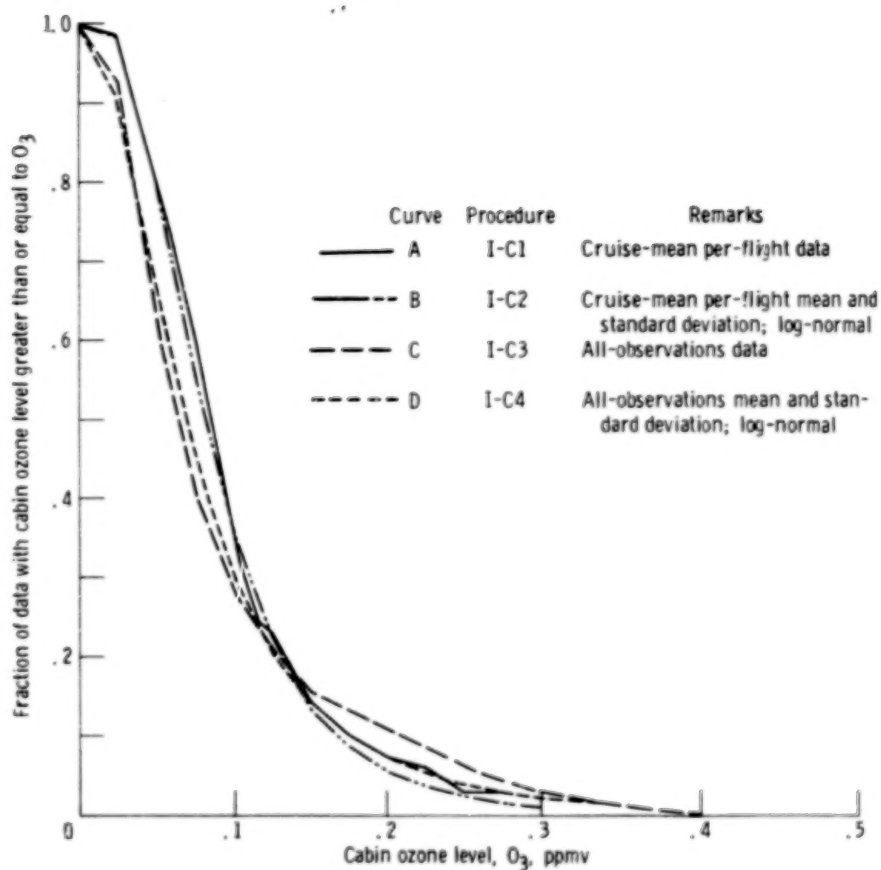


Figure 1. - Cabin ozone cumulative frequency distributions for problem I example-case solutions. Boeing 747-100 in domestic U. S. service from 3/30/77 to 6/13/77.

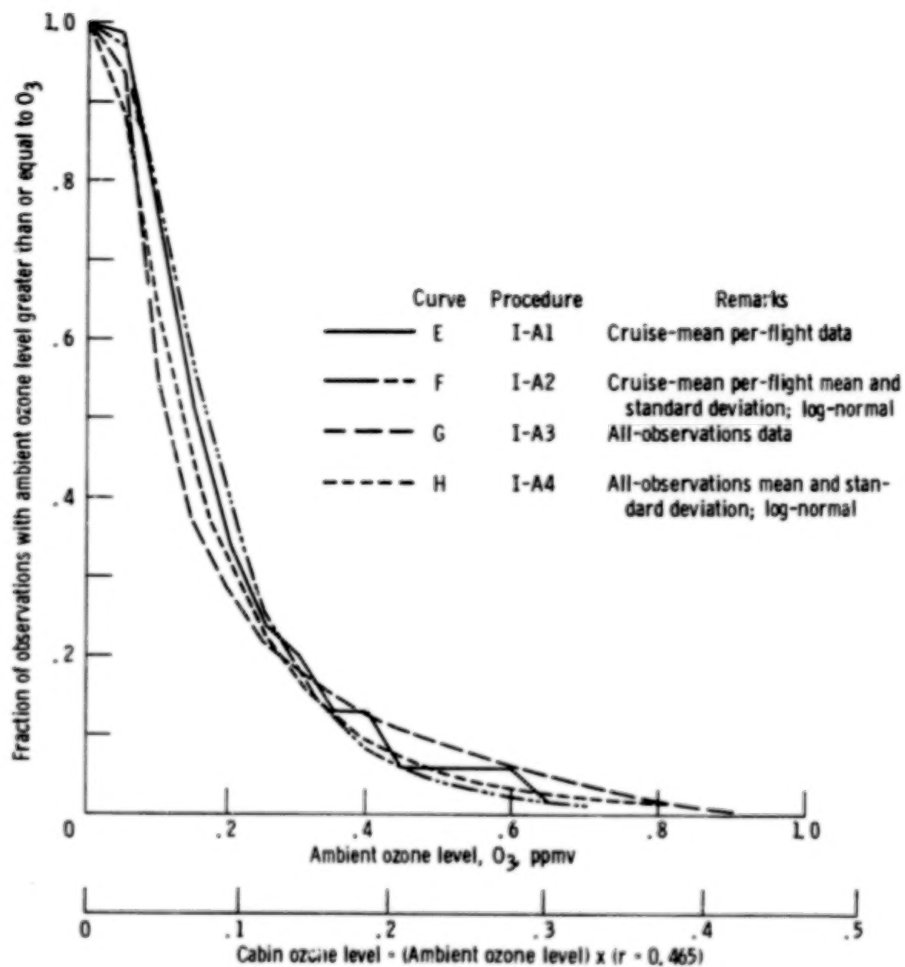


Figure 2 - Ambient ozone cumulative frequency distributions for problem I example-case solutions. Boeing 747-100 in domestic U. S. service from 3/30/77 to 6/30/77.

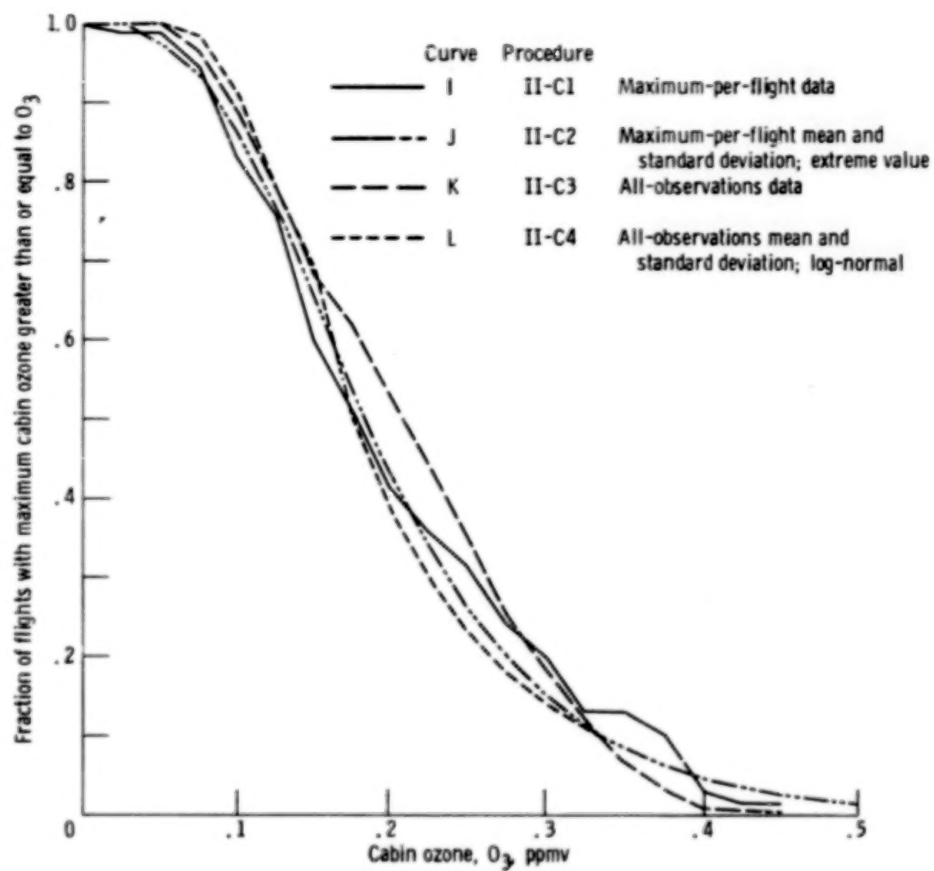


Figure 3. - Cabin ozone maximum-per-flight cumulative frequency distributions for problem II example-case solutions, Boeing 747-100 in domestic U. S. service from 3/30/77 to 6/13/77.



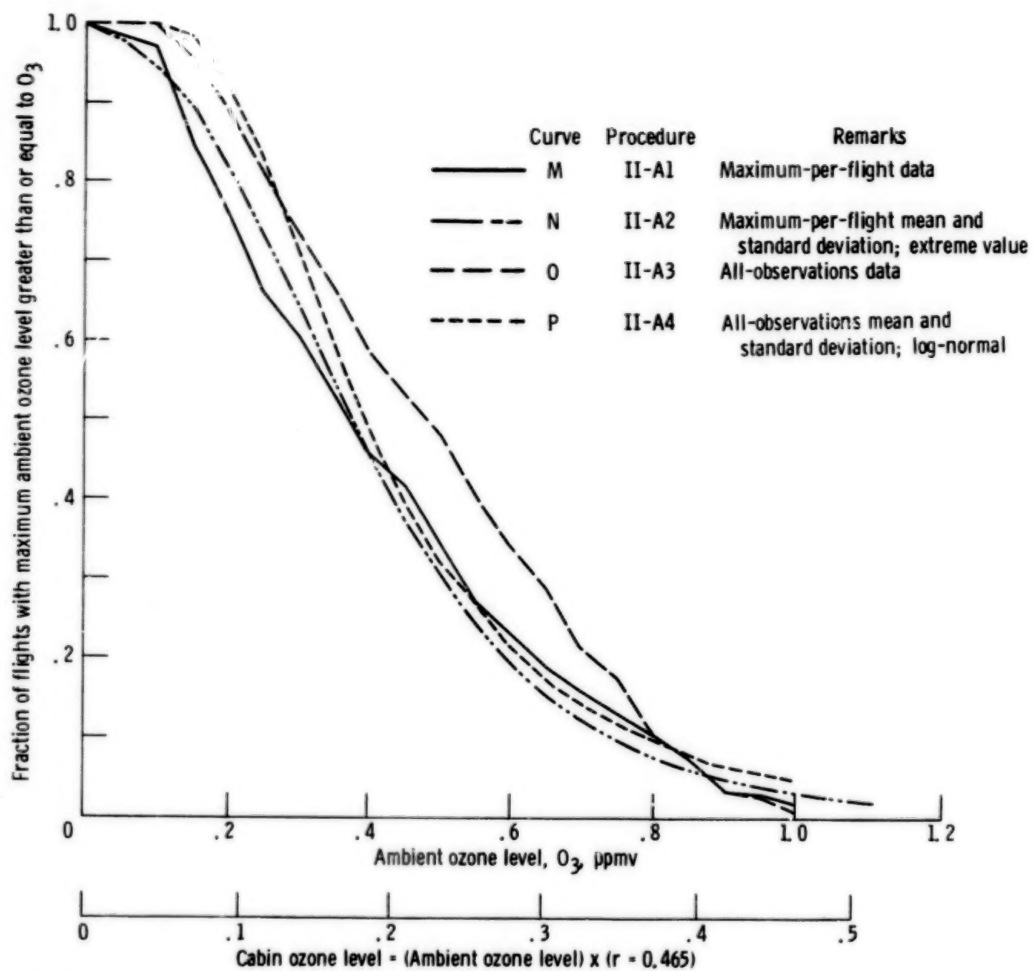


Figure 4 - Ambient ozone maximum-per-flight cumulative frequency distributions for problem II example-case solutions. Boeing 747-100 in domestic U. S. service from 3/30/77 to 6/13/77.

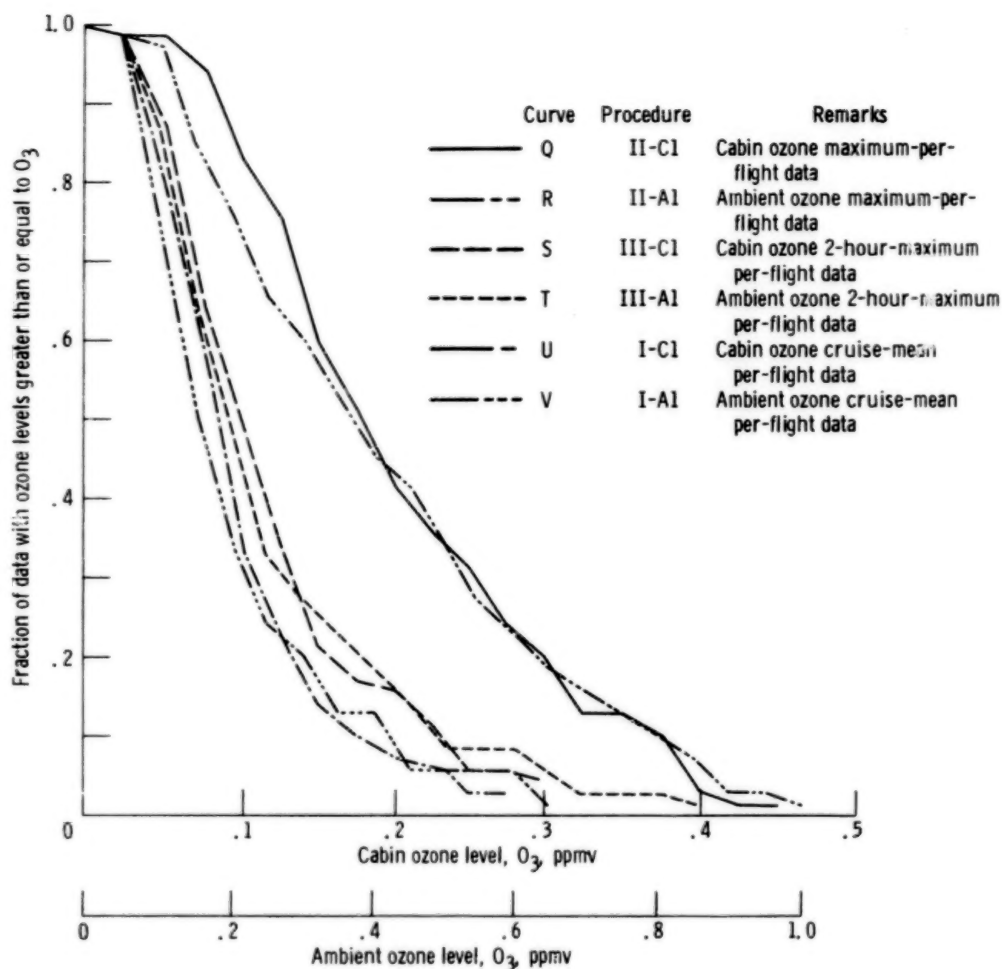


Figure 5. - More ozone cumulative frequency distributions for example-case solutions. Boeing 747-100 in domestic U. S. service from 3/30/77 to 6/13/77.

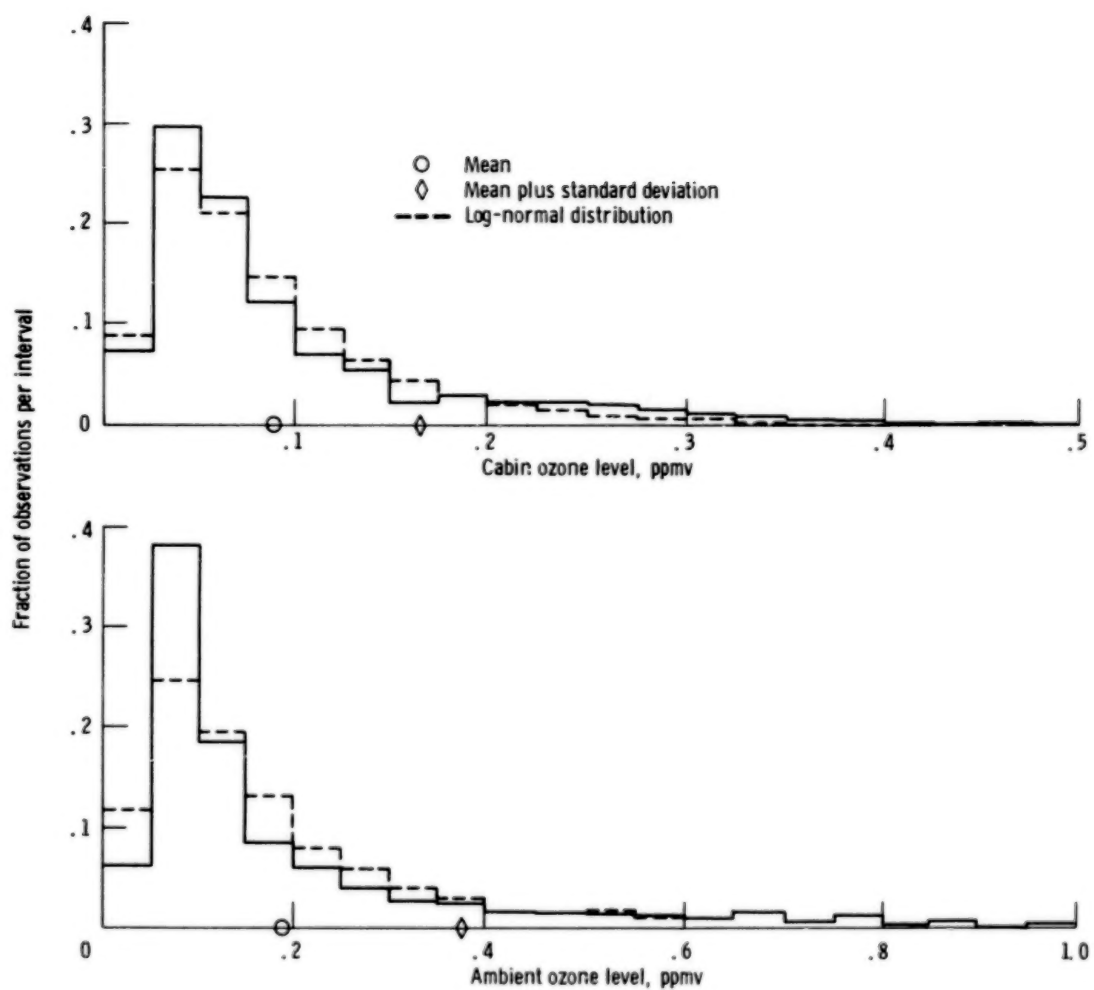


Figure 6. - Frequency distribution of ozone data from example-case data set.

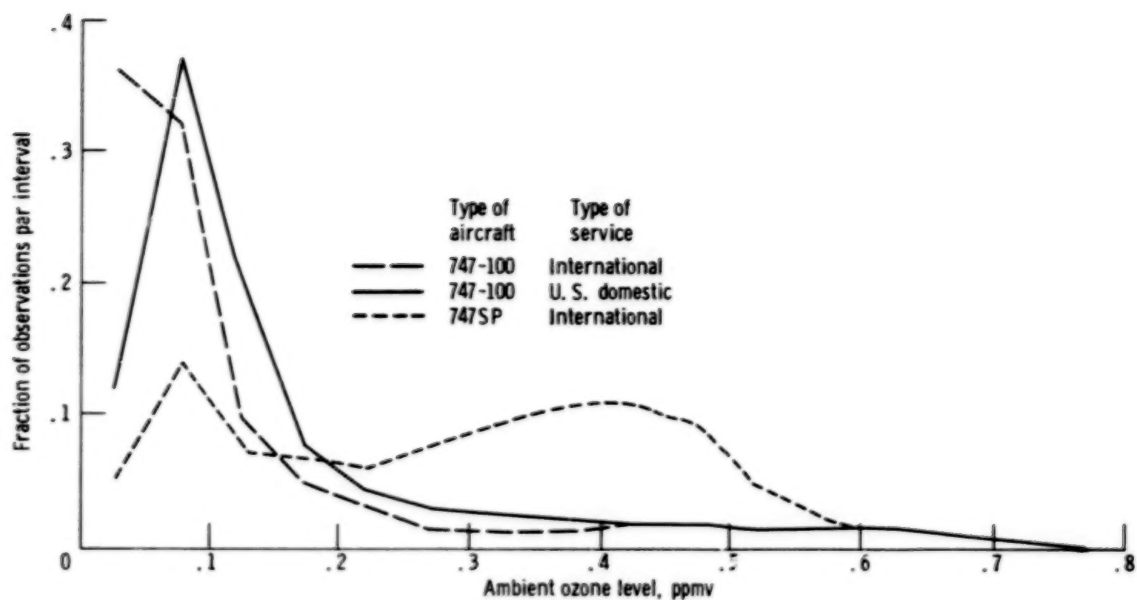


Figure 7. - Ambient ozone frequency distribution for second quarters (April, May, June) of 1975 and 1976.

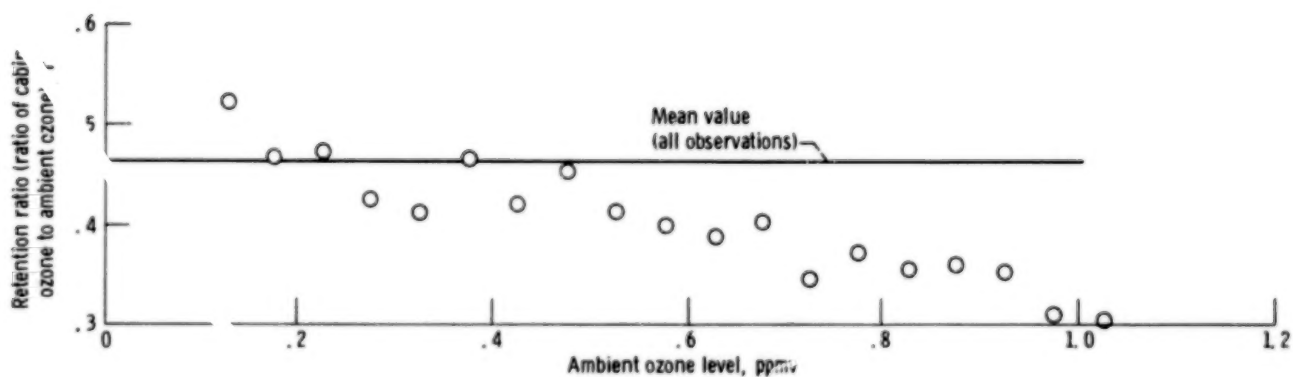
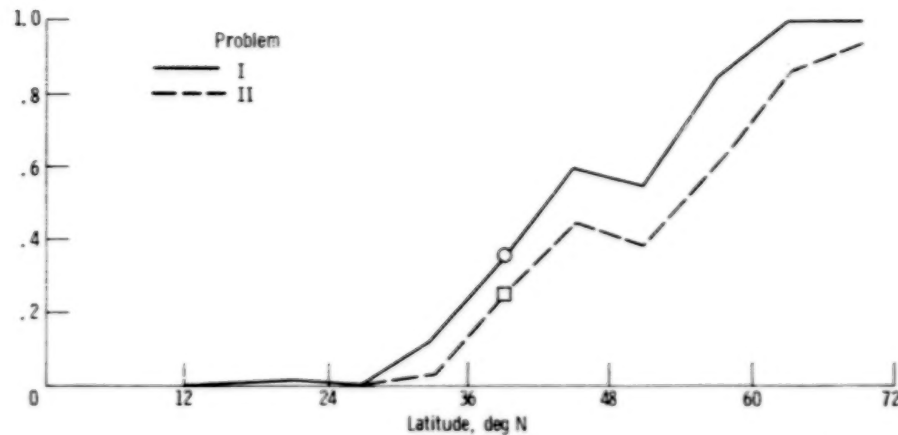
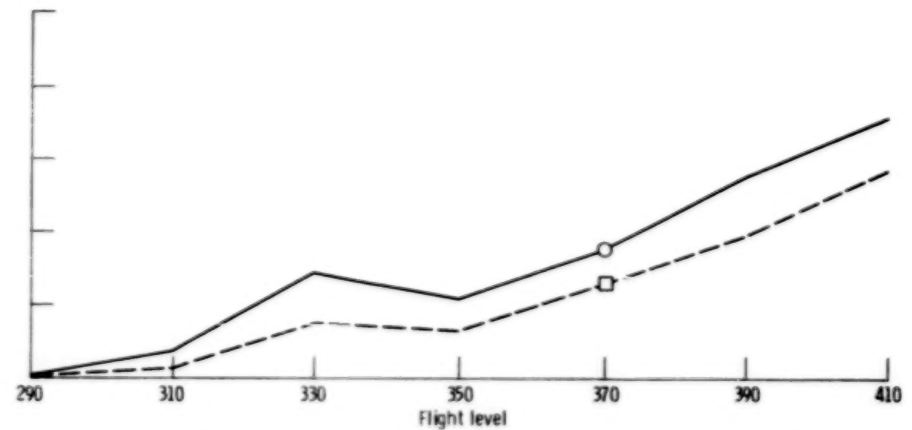


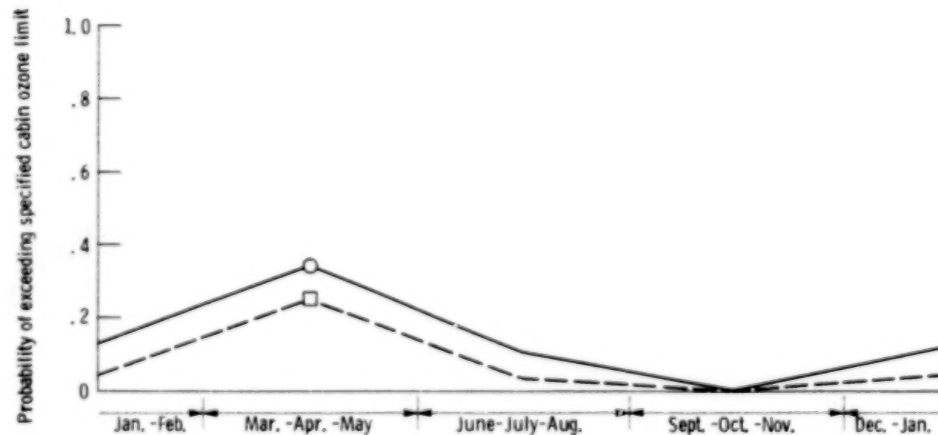
Figure 8. - Variation of retention ratio with ambient ozone level for example-case data.



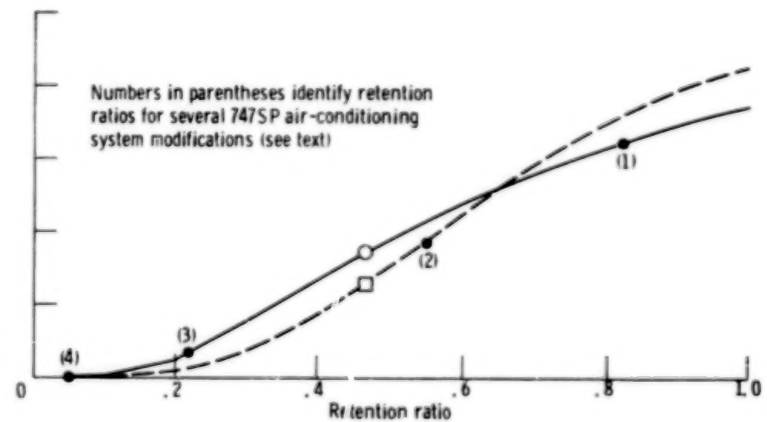
(a) Flight level, 370; March to May; retention ratio, 0.465; independent observations per flight, 6.7; flight-segment-average limit, 0.1 ppmv; maximum-per-flight limit, 0.3 ppmv.



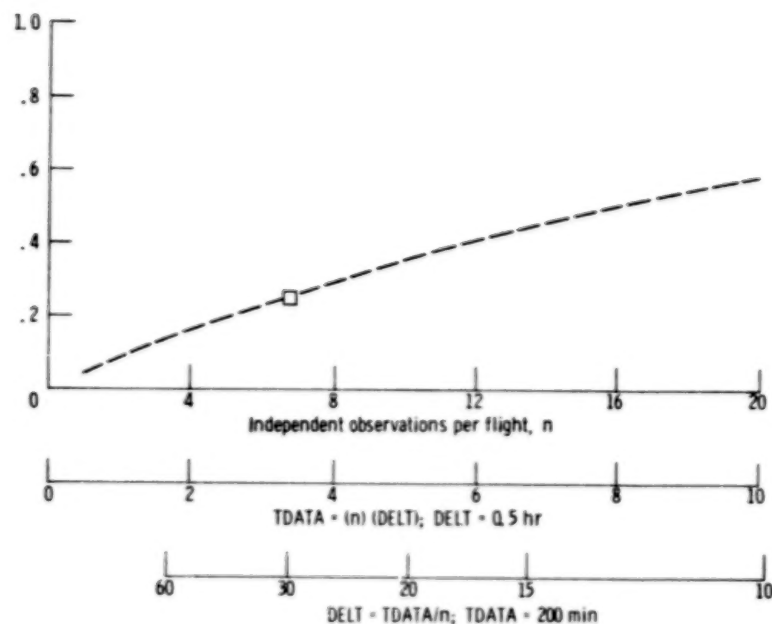
(b) Latitude,  $36^{\circ}$  to  $42^{\circ}$  N; March to May; retention ratio, 0.465; independent observations per flight, 6.7; flight-segment-average limit, 0.1 ppmv; maximum-per-flight limit, 0.3 ppmv.



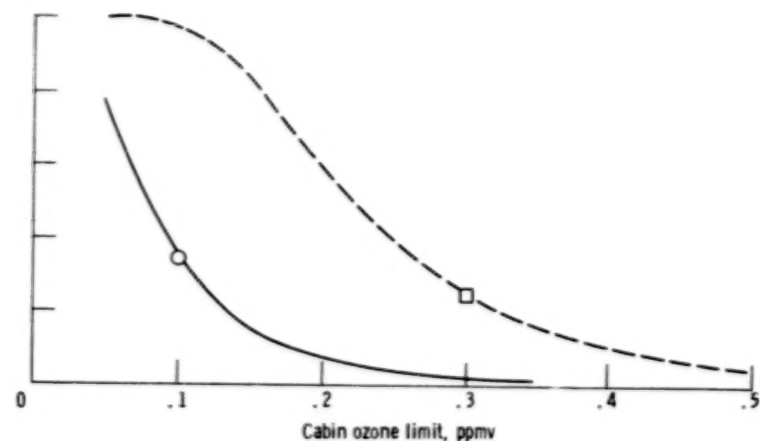
(c) Latitude,  $36^{\circ}$  to  $42^{\circ}$  N; flight level, 370; retention ratio, 0.465; independent observations per flight, 6.7; flight-segment-average limit, 0.1 ppmv; maximum-per-flight limit, 0.3 ppmv.



(d) Latitude,  $36^{\circ}$  to  $42^{\circ}$  N; flight level, 370; March to May; independent observations per flight, 6.7; flight-segment-average limit, 0.1 ppmv; maximum-per-flight limit, 0.3 ppmv.



(e) Latitude,  $36^\circ$  to  $42^\circ$  N; flight level, 370; March to May; retention ratio, 0.465; flight-segment-average limit, 0.1 ppmv; maximum-per-flight limit, 0.3 ppmv.



(f) Latitude,  $36^\circ$  to  $42^\circ$  N; flight level, 370; March to May; retention ratio, 0.465; independent observations per flight, 6.7.

Figure 9. - Effect on estimated probabilities of variations in independent variables.

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16. Abstract <p>Three analytical problems in estimating the frequency at which commercial airline flights will encounter high cabin ozone levels are formulated and solved: namely, estimating flight-segment-mean levels, estimating maximum-per-flight levels, and estimating the maximum average level over a specified flight interval. For each problem, solution procedures are given for different levels of input information - from complete cabin ozone data, which provides a direct solution, to limited ozone information, such as ambient ozone means and standard deviations, with which several assumptions are necessary to obtain the required estimates. Each procedure is illustrated by an example-case calculation that uses simultaneous cabin and ambient ozone data obtained by the NASA Global Atmospheric Sampling Program (GASP). Critical assumptions are discussed and evaluated, and the several solutions for each problem are compared. Example calculations are also performed to illustrate how variations in latitude, altitude, season, retention ratio, flight duration, and cabin ozone limits affect the estimated probabilities.</p>			
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